Unit 9

Implementing Combinational Functions with Karnaugh Maps

Outcomes

- I can use Karnaugh maps to synthesize combinational functions with several outputs
- I can determine the appropriate size and contents of a memory to implement any logic function (i.e. truth table)

Covering Combinations

- A minterm corresponds to ("covers") 1 combination of a logic function
- As we __________ variables from a product term, more combinations are covered
  - The product term will evaluate to true __________ of the removed variables value (i.e. the term is independent of that variable)

F = WX'YZ
= m11

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**0**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**0**

F = WX'Z
= m9+m11

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**0**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**0**

Covering Combinations

- The more variables we can remove the more ________________ a single product term covers
  - Said differently, a small term will cover (or expand to) more combinations
- The smaller the term, the smaller the __________
  - We need fewer ________ to check for multiple combinations
- For a given function, how can we find these smaller terms?

F = X'Z
= m1+m3+m9+m11

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**0**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**0**

F = X'
= m0+m1+m2+m3+m8+m9+m10+m11

W	X	Y	Z	F
0	0	0	0	**1**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**0**

A new way to synthesize your logic functions

KARNAUGH MAPS

Logic Function Synthesis

- Given a function description as a T.T. or sum of minterm (product of maxterm) form, how can we arrive at a circuit implementation or equation (i.e. perform logic synthesis)?
- Methods
  - Minterms / maxterms
    - Use ___________________ to find minimal 2-level implementation
  - Karnaugh Maps [we will learn this one now]
    - Graphical method amenable to human ____________ inspection and can be used for functions of **up to ___ variables (but becomes large and unwieldy after just ____ variables)**
  - Quine-McCluskey Algorithm (amenable to computer implementations
  - Others: Espresso algorithm, Binary Decision Diagrams, etc.

Karnaugh Maps

- If used correctly, will always yield a minimal, ____________ implementation
  - There may be a more minimal 3-level, 4-level, 5-level… implementation but K-maps produce the minimal two-level (SOP or POS) implementation
- Represent the truth table graphically as a series of adjacent __________ that allows a human to see where variables can be removed

Gray Code

- Different than normal binary ordering
- Reflective code
  - When you add the $(n+1)^{th}$ bit, reflect all the previous n-bit combinations
- Consecutive code words differ by only 1-bit

differ by only 1-bit

0	0
0	1
1	1
1	0

2-bit Gray code

when you move to the next bit, reflect the previous combinations

0	0	0
0	0	1
0	1	1
0	1	0

differ by only 1-bit

differ by only 1-bit

3-bit Gray code

Karnaugh Map Construction

- Every square represents 1 input combination
- Must label axes in Gray code order
- Fill in squares with given function values

X	Y	Z	F
0	0	0	**0**
0	0	1	**1**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**0**

F(x,y,z)=m1 + m4 + m5 + m6

Z \ XY	00	01	11	10
0	[0] 0	[2] 0	[6] 1	[4] 1
1	[1] 1	[3] 0	[7] 0	[5] 1

3 Variable Karnaugh Map

G(w,x,y,z)=m1+m2+m3+m5+m6+m7+m9+ m10+m11+m14+m15

YZ \ WX	00	01	11	10
00	[0] 0	[4] 0	[12] 0	[8] 0
01	[1] 1	[5] 1	[13] 0	[9] 1
11	[3] 1	[7] 1	[15] 1	[11] 1
10	[2] 1	[6] 1	[14] 1	[10] 1

4 Variable Karnaugh Map

Karnaugh Maps

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	[0] 0	[4] 0	[12] 0	[8] 0
01	[1] 1	[5] 1	[13] 0	[9] 1
11	[3] 1	[7] 1	[15] 1	[11] 1
10	[2] 1	[6] 1	[14] 1	[10] 1

Karnaugh Maps

- Squares with a '1' represent minterms that must be included in the SOP solution
- Squares with a '0' represent maxterms that must be included in the POS solution

YZ \ WX	00	01	11	10
00	0 0	4 0	12 0	8 0
01	1 1	5 1	13 0	9 1
11	3 1	7 1	15 1	11 1
10	2 1	6 1	14 1	10 1

Maxterm:
w' + x' + y + z

Maxterm:
w' + x + y + z

Minterm:
w•x'•y•z

Minterm:
w•x'•y•z'

Karnaugh Maps

- Groups (of 2, 4, 8, etc.) of adjacent 1's will always simplify to smaller product term than just individual minterms

F=m0+m2+m4+m5+m6

Z \ XY	00	01	11	10
0	^{0} 1	^{2} 1	^{6} 1	^{4} 1
1	^{1} 0	^{3} 0	^{7} 0	^{5} 1

3 Variable Karnaugh Map

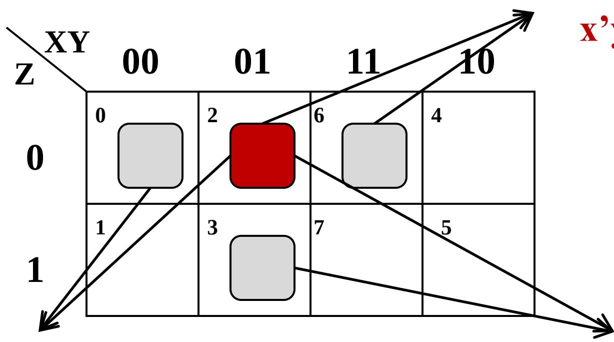
Karnaugh Maps

- Adjacent squares differ by 1-variable
  - This will allow us to use T10 = AB + AB'= A or T10' = (A+B')(A+B) = A

3 Variable Karnaugh Map

Difference in X: 010 & 110

x'yz' + xyz'
= yz'



Difference in Y: 010 & 000

x'yz' + x'y'z'
= x'z'

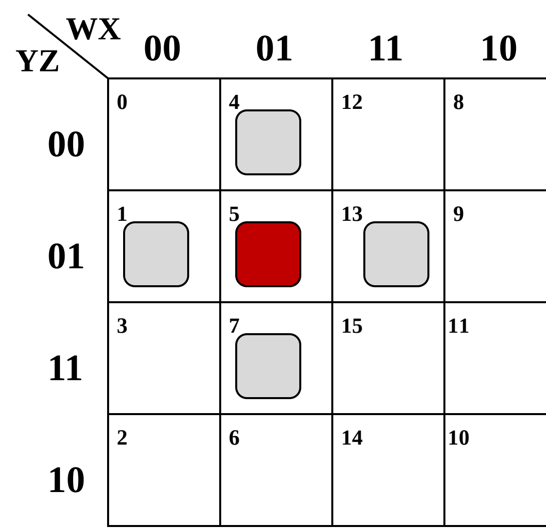
Difference in Z: 010 & 011

x'yz' + x'yz
= x'y

0 = 0<u>0</u>0
2 = <u>010</u>
3 = 01<u>1</u>
6 = <u>1</u>10

Adjacent squares
differ by 1-bit

4 Variable Karnaugh Map



1 = 0<u>0</u>01
4 = 010<u>0</u>
5 = <u>0101</u>
7 = 01<u>1</u>1
13 = <u>1</u>101

Adjacent squares
differ by 1-bit

Karnaugh Maps

- 2 adjacent 1’s (or 0’s) differ by only one variable
- 4 adjacent 1’s (or 0’s) differ by two variables
- 8, 16, … adjacent 1’s (or 0’s) differ by 3, 4, … variables
- By grouping adjacent squares with 1’s (or 0’s) in them, we can come up with a simplified expression using T10 (or T10’ for 0’s)

YZ \ WX	00	01	11	10
00	$_{0}$ 0	$_{4}$ 0	$_{12}$ 0	$_{8}$ 0
01	$_{1}$ 1	$_{5}$ 1	$_{13}$ 0	$_{9}$ 1
11	$_{3}$ 1	$_{7}$ 1	$_{15}$ 1	$_{11}$ 1
10	$_{2}$ 1	$_{6}$ 1	$_{14}$ 1	$_{10}$ 1

$w'\bullet x'\bullet y'\bullet z + w'\bullet x'\bullet y\bullet z + w'\bullet x\bullet y'\bullet z + w'\bullet x\bullet y\bullet z$

$= \mathbf{w'\bullet z}$

w’z are constant while all combos of x and y are present (x’y’, x’y, xy’, xy)

$\mathbf{(w'+x'+y+z)\bullet(w'+x'+y+z') = (w'+x'+y)}$

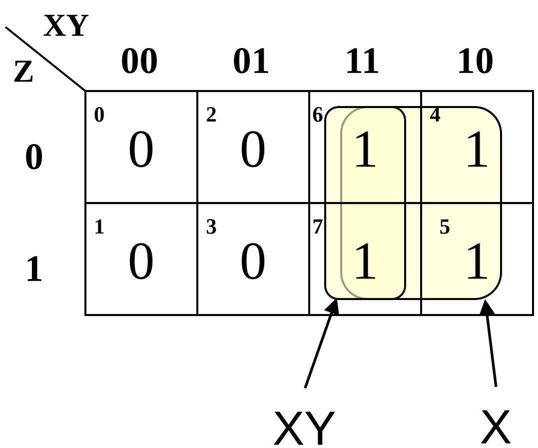
$\mathbf{w\bullet x\bullet y\bullet z + w\bullet x'\bullet y\bullet z = w\bullet y\bullet z}$

K-Map View of the Theorems

- The 2 & 3 variable theorems used to simplify expressions can be illustrated using K-Maps.

T9: Covering

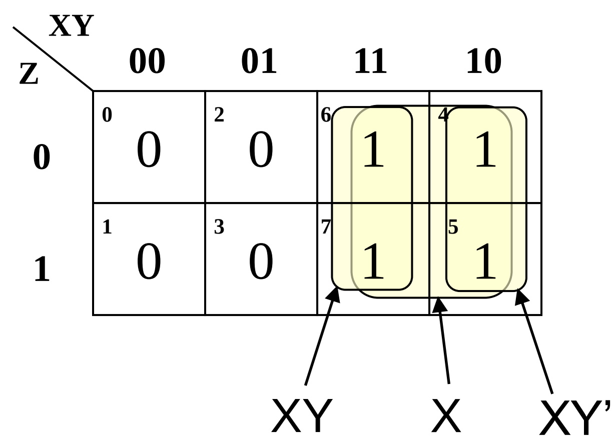
X + XY = X



X "covers" XY so XY not needed

T10: Combining

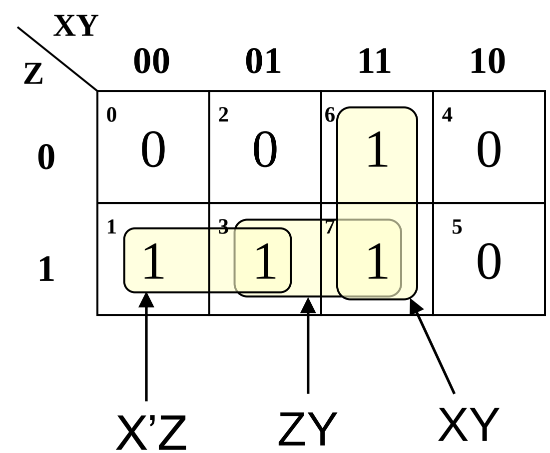
XY + XY' = X



XY and XY' can be combined to form X

T11: Consensus

XY + X'Z + ZY = XY + X'Z



Don't need ZY if you have X'Z and XY

K-Map Grouping Rules

- Cover the 1's [=on-set] or 0's [=off-set] with _______ groups as possible, but make those groups _________ as possible
  - Make them as large as possible even if it means "covering" a 1 (or 0) that's already a member of another group
- Make groups of ______________, ... and they must be rectangular or square in shape.
- Wrapping is legal

Group These K-Maps

Z \ XY	00	01	11	10
0	[0] 0	[2] 1	[6] 0	[4] 0
1	[1] 1	[3] 0	[7] 0	[5] 0

Z \ XY	00	01	11	10
0	[0] 1	[2] 1	[6] 0	[4] 0
1	[1] 1	[3] 0	[7] 0	[5] 0

YZ \ WX	00	01	11	10
00	[0] 0	[4] 0	[12] 1	[8] 1
01	[1] 1	[5] 1	[13] 1	[9] 0
11	[3] 1	[7] 1	[15] 1	[11] 0
10	[2] 0	[6] 0	[14] 0	[10] 1

Karnaugh Maps

YZ \ WX	00	01	11	10
00	(0) 0	(4) 1	(12) 1	(8) 1
01	(1) 0	(5) 1	(13) 1	(9) 1
11	(3) 0	(7) 1	(15) 1	(11) 1
10	(2) 0	(6) 0	(14) 1	(10) 1

- Cover the remaining ‘1’ with the largest group possible even if it “reuses” already covered 1’s

Karnaugh Maps

- Groups can wrap around from:
  - Right to left
  - Top to bottom
  - Corners

YZ \ WX	00	01	11	10
00	0: 0	4: 0	12: 1	8: 0
01	1: 1	5: 0	13: 0	9: 1
11	3: 1	7: 0	15: 0	11: 1
10	2: 0	6: 0	14: 1	10: 0

F = X'Z + WXZ'

YZ \ WX	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 0	5: 0	13: 0	9: 0
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 0	10: 1

F = X'Z'

Group This

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 0	(8) 0
01	(1) 1	(5) 1	(13) 0	(9) 1
11	(3) 1	(7) 1	(15) 1	(11) 1
10	(2) 1	(6) 1	(14) 1	(10) 1

K-Map Translation Rules

- When translating a group of 1's, find the variable values that are constant for each square in the group and translate only those variables values to a product term
- Grouping 1's yields SOP
- When translating a group of 0's, again find the variable values that are constant for each square in the group and translate only those variable values to a sum term
- Grouping 0's yields POS

Karnaugh Maps (SOP)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 0	(8) 0
01	(1) 1	(5) 1	(13) 0	(9) 1
11	(3) 1	(7) 1	(15) 1	(11) 1
10	(2) 1	(6) 1	(14) 1	(10) 1

F =

Karnaugh Maps (SOP)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

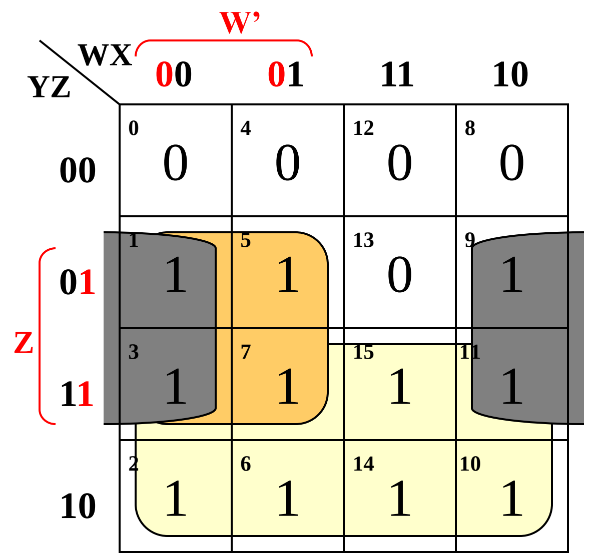
YZ \ WX	00	01	11	10
00	^{0} 0	^{4} 0	^{12} 0	^{8} 0
01	^{1} 1	^{5} 1	^{13} 0	^{9} 1
11	^{3} 1	^{7} 1	^{15} 1	^{11} 1
10	^{2} 1	^{6} 1	^{14} 1	^{10} 1

Y

F = Y

Karnaugh Maps (SOP)

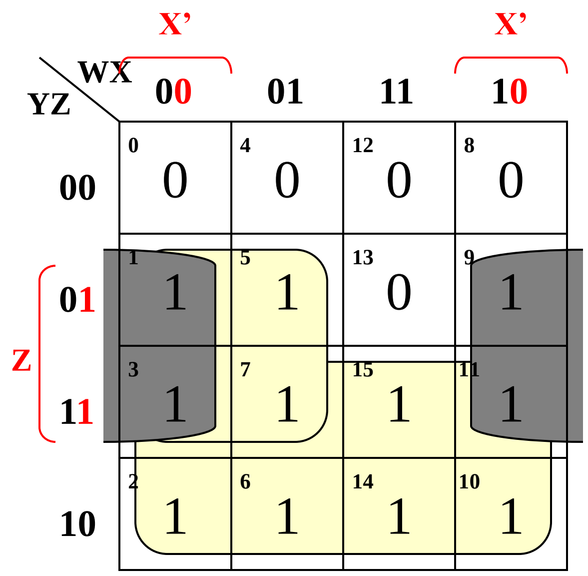
W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



F = Y + W'Z + …

Karnaugh Maps (SOP)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



F = Y + W'Z + X'Z

Karnaugh Maps (POS)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 0	(8) 0
01	(1) 1	(5) 1	(13) 0	(9) 1
11	(3) 1	(7) 1	(15) 1	(11) 1
10	(2) 1	(6) 1	(14) 1	(10) 1

F =

Karnaugh Maps (POS)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
Y,Z 00	(0) 0	(4) 0	(12) 0	(8) 0
01	(1) 1	(5) 1	(13) 0	(9) 1
11	(3) 1	(7) 1	(15) 1	(11) 1
10	(2) 1	(6) 1	(14) 1	(10) 1

F = (Y+Z)

Karnaugh Maps (POS)

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

WX

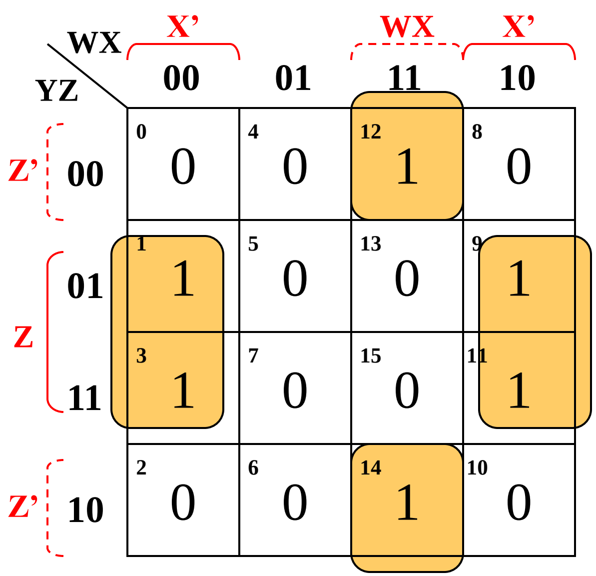
Y

YZ \ WX	00	01	11	10
00	$_{0}$ 0	$_{4}$ 0	$_{12}$ 0	$_{8}$ 0
01	$_{1}$ 1	$_{5}$ 1	$_{13}$ 0	$_{9}$ 1
11	$_{3}$ 1	$_{7}$ 1	$_{15}$ 1	$_{11}$ 1
10	$_{2}$ 1	$_{6}$ 1	$_{14}$ 1	$_{10}$ 1

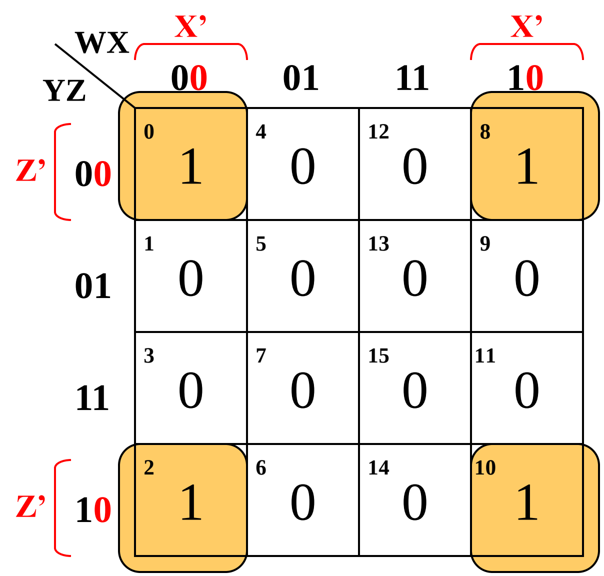
F = (Y+Z)(W'+X'+Y)

Karnaugh Maps

- Groups can wrap around from:
  - Right to left
  - Top to bottom
  - Corners



F = X’Z + WXZ’



F = X’Z’

Exercises

YZ \ WX	00	01	11	10
00	^{0} 1	^{4} 0	^{12} 0	^{8} 1
01	^{1} 1	^{5} 0	^{13} 0	^{9} 1
11	^{3} 0	^{7} 0	^{15} 0	^{11} 0
10	^{2} 1	^{6} 0	^{14} 1	^{10} 1

$F_{SOP}=$

YZ \ WX	00	01	11	10
00	^{0} 1	^{4} 0	^{12} 0	^{8} 1
01	^{1} 1	^{5} 0	^{13} 0	^{9} 1
11	^{3} 0	^{7} 0	^{15} 0	^{11} 0
10	^{2} 1	^{6} 0	^{14} 1	^{10} 1

$F_{POS}=$

P(x,y,z)=m2+m3+m5+m7

P=

No Redundant Groups

YZ \ WX	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: 1
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: 1

This group does not cover new squares that are not already part of another essential grouping

Multiple Minimal Expressions

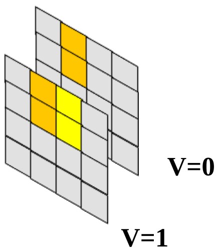
- For some functions, ___________________ groupings exist which will lead to alternate minimal __________________…Pick one

D2D1 \ D8D4	00	01	11	10
00	^{0} 0	^{4} 0	^{12} 1	^{8} 1
01	^{1} 0	^{5} 0	^{13} 1	^{9} 1
11	^{3} 1	^{7} 1	^{15} 1	^{11} 0
10	^{2} 1	^{6} 1	^{14} 0	^{10} 0

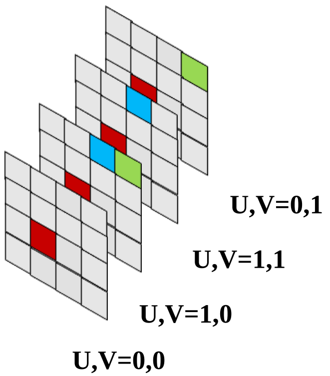
Best way to cover this ‘1’??

Karnaugh Maps Beyond 4 Variables

- Recall, K-Maps require an adjacency for each variable
  - To see the necessary adjacencies, 5 and 6 variable K-Maps can be thought of in three dimensions
- Can we have 7-variable K-Maps?
  - No! We would need to see 7 adjacencies per square and we humans cannot visualize 4 dimensions
- Other computer-friendly minimization algorithms
  - Quine-McCluskey
    - Still exponential runtime
    - Minimization is NP-hard problem
  - Espresso-heuristic Minimizer
    - Achieves "good" minimization in far less time (may not be absolute minimal)



5 Variable K-Maps



6 Variable K-Maps

DON'T CARE OUTPUTS

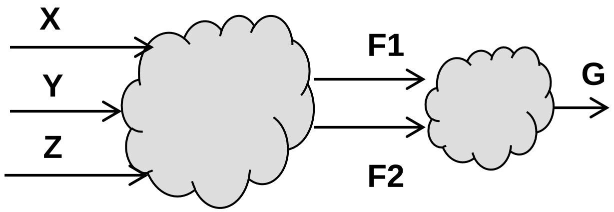
Don't-Cares

- Sometimes there are certain input combinations that are illegal (due to physical or other external constraints)
- The outputs for the illegal inputs are "don't-cares"
  - The output can either be 0 or 1 since the inputs can never occur
  - Don't-cares can be included in groups of 1 or groups of 0 when grouping in K-Maps
  - Use them to make as big of groups as possible

Use 'Don't care' outputs as wildcards (e.g. the blank tile in Scrabble™). They can be either 0 or 1 whatever helps make bigger groups to cover the ACTUAL 1's

Invalid Input Combinations

- Given intermediate functions F1 and F2, how could you use AND, OR, NOT to make G
- Notice certain F1,F2 combinations never occur in G(x,y,z)…what should we make their output in the T.T.



X	Y	Z	F1	F2	G
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	1	0

F1	F2	G
0	0	
0	1	
1	0	
1	1	

Invalid Input Combinations

- An example of where Don't-Cares may come into play is Binary Coded Decimal (BCD)
  - Rather than convert a decimal number to unsigned binary (i.e. summing increasing powers of 2) we can represent each decimal digit as a separate group of 4-bits (with weights 8,4,2,1 for each group of 4 bits)
  - **Combinations 1010-1111 cannot occur!**

$(439)_{10}$

BCD Representation: 0100 0011 1001

8 4 2 1 8 4 2 1 8 4 2 1

This is not the binary representation of 439, it is the Binary Coded Decimal (BCD) representation

Important: BCD represent each decimal digit with a <u>separate</u> group of bits

Don’t Care Example

D8	D4	D2	D1	GT6
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**d**
1	0	1	1	**d**
1	1	0	0	**d**
1	1	0	1	**d**
1	1	1	0	**d**
1	1	1	1	**d**

D2D1 \ D8D4	00	01	11	10
00	(0) 0	(4) 0	(12) d	(8) 1
01	(1) 0	(5) 0	(13) d	(9) 1
11	(3) 0	(7) 1	(15) d	(11) d
10	(2) 0	(6) 0	(14) d	(10) d

$GT6_{SOP}=$

D2D1 \ D8D4	00	01	11	10
00	(0) 0	(4) 0	(12) d	(8) 1
01	(1) 0	(5) 0	(13) d	(9) 1
11	(3) 0	(7) 1	(15) d	(11) d
10	(2) 0	(6) 0	(14) d	(10) d

$GT6_{POS}=$

Don’t Cares

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**d**
1	0	1	1	**d**
1	1	0	0	**d**
1	1	0	1	**d**
1	1	1	0	**d**
1	1	1	1	**d**

Reuse “d’s” to make as large a group as possible to cover 1,5, & 9

YZ \ WX	00	01	11	10
00	0: 0	4: 0	12: d	8: 0
01	1: 1	5: 1	13: d	9: 1
11	3: 1	7: 1	15: d	11: d
10	2: 1	6: 1	14: d	10: d

F = Z + Y

Use these 4 “d’s” to make a group of 8

Don’t Cares

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**d**
1	0	1	1	**d**
1	1	0	0	**d**
1	1	0	1	**d**
1	1	1	0	**d**
1	1	1	1	**d**

You can use “d’s” when grouping 0’s and converting to POS

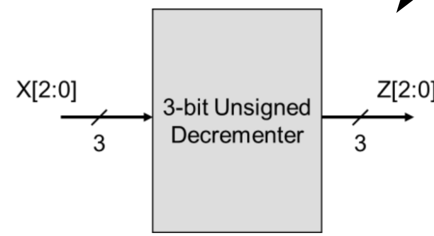
YZ \ WX	00	01	11	10
00	^{0} 0	^{4} 0	^{12} d	^{8} 0
01	^{1} 1	^{5} 1	^{13} d	^{9} 1
11	^{3} 1	^{7} 1	^{15} d	^{11} d
10	^{2} 1	^{6} 1	^{14} d	^{10} d

F = Y+Z

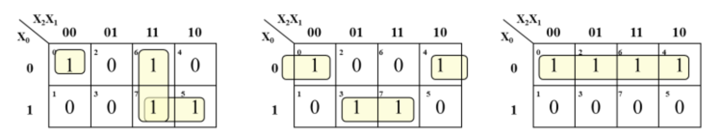
A GENERAL, COMBINATIONAL CIRCUIT DESIGN PROCESS

Combinational Design Process

- Understand the problem
  - How many input bits and their representation system
  - How many output bits need be generated and what are their representation
  - Draw a block diagram
- Write a truth table
- Use a K-map to derive an equation for EACH output bit
- Use the equation to draw a circuit for EACH output bit, letting each circuit run in parallel to produce their respective output bit



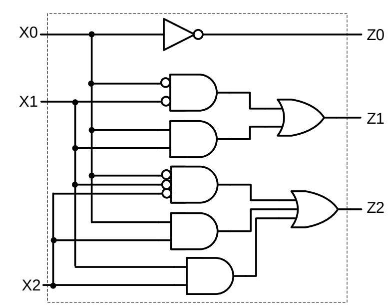
X_2	X_1	X_0	Z_2	Z_1	Z_0
0	0	0	**1**	**1**	**1**
0	0	1	**0**	**0**	**0**
0	1	0	**0**	**0**	**1**
0	1	1	**0**	**1**	**0**
1	0	0	**0**	**1**	**1**
1	0	1	**1**	**0**	**0**
1	1	0	**1**	**0**	**1**
1	1	1	**1**	**1**	**0**



$Z_2 = X_2X_0 + X_2X_1 + X_2'X_1'X_0'$

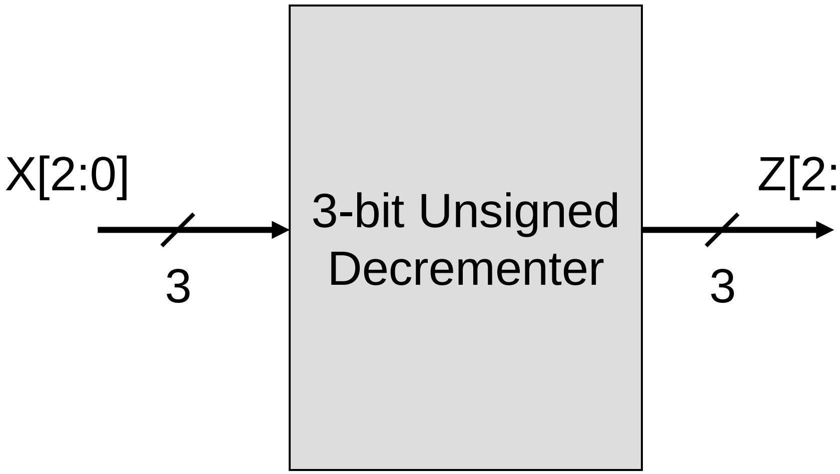
$Z_1 = X_1'X_0' + X_1X_0$

$Z_0 = X_0'$



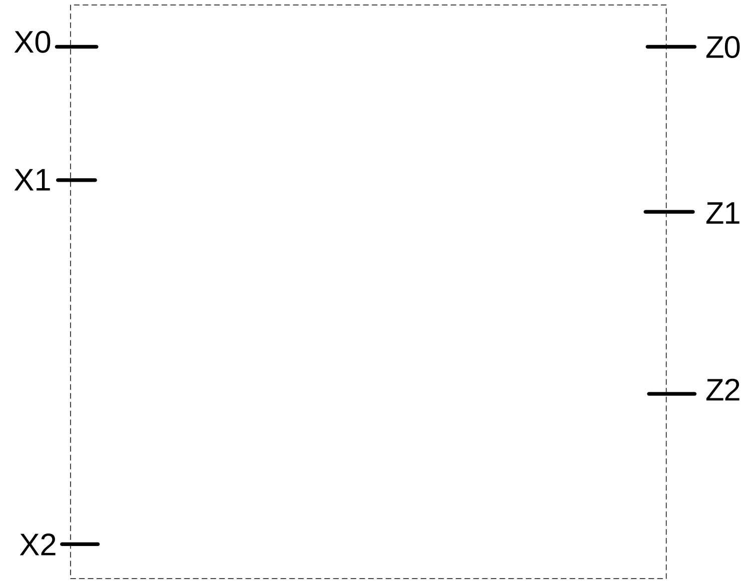
Designing Circuits w/ K-Maps

- Given a description…
  - Block Diagram
  - Truth Table
  - K-Map for each output bit (each output bit is a separate function of the inputs)
- 3-bit unsigned decrementer (Z = X-1)
  - If X[2:0] = 000 then Z[2:0] = 111, etc.



3-bit Number Decrementer

X_2	X_1	X_0	Z_2	Z_1	Z_0
0	0	0	**1**	**1**	**1**
0	0	1	**0**	**0**	**0**
0	1	0	**0**	**0**	**1**
0	1	1	**0**	**1**	**0**
1	0	0	**0**	**1**	**1**
1	0	1	**1**	**0**	**0**
1	1	0	**1**	**0**	**1**
1	1	1	**1**	**1**	**0**



X_0 \ X_2X_1	00	01	11	10
0	1	0	1	0
1	0	0	1	1

$Z_2 = X_2X_0 + X_2X_1 + X_2'X_1'X_0'$

X_0 \ X_2X_1	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$Z_1 = X_1'X_0' + X_1X_0$

X_0 \ X_2X_1	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$Z_0 = X_0'$

Squaring Circuit

- Design a combinational circuit that accepts a 3-bit number and generates an output binary number equal to the square of the input number. ($B = A^2$)
- Using 3 bits we can represent the numbers from _______ to ______ .
- The possible squared values range from _______ to _______ .
- Thus to represent the possible outputs we need how many bits? ________

3-bit Squaring Circuit

	Inputs			Outputs						
A	A_2	A_1	A_0	B_5	B_4	B_3	B_2	B_1	B_0	$B=A^2$

A0 \ A2A1	00	01	11	10
0	0	2	6	4
1	1	3	7	5

B5 =

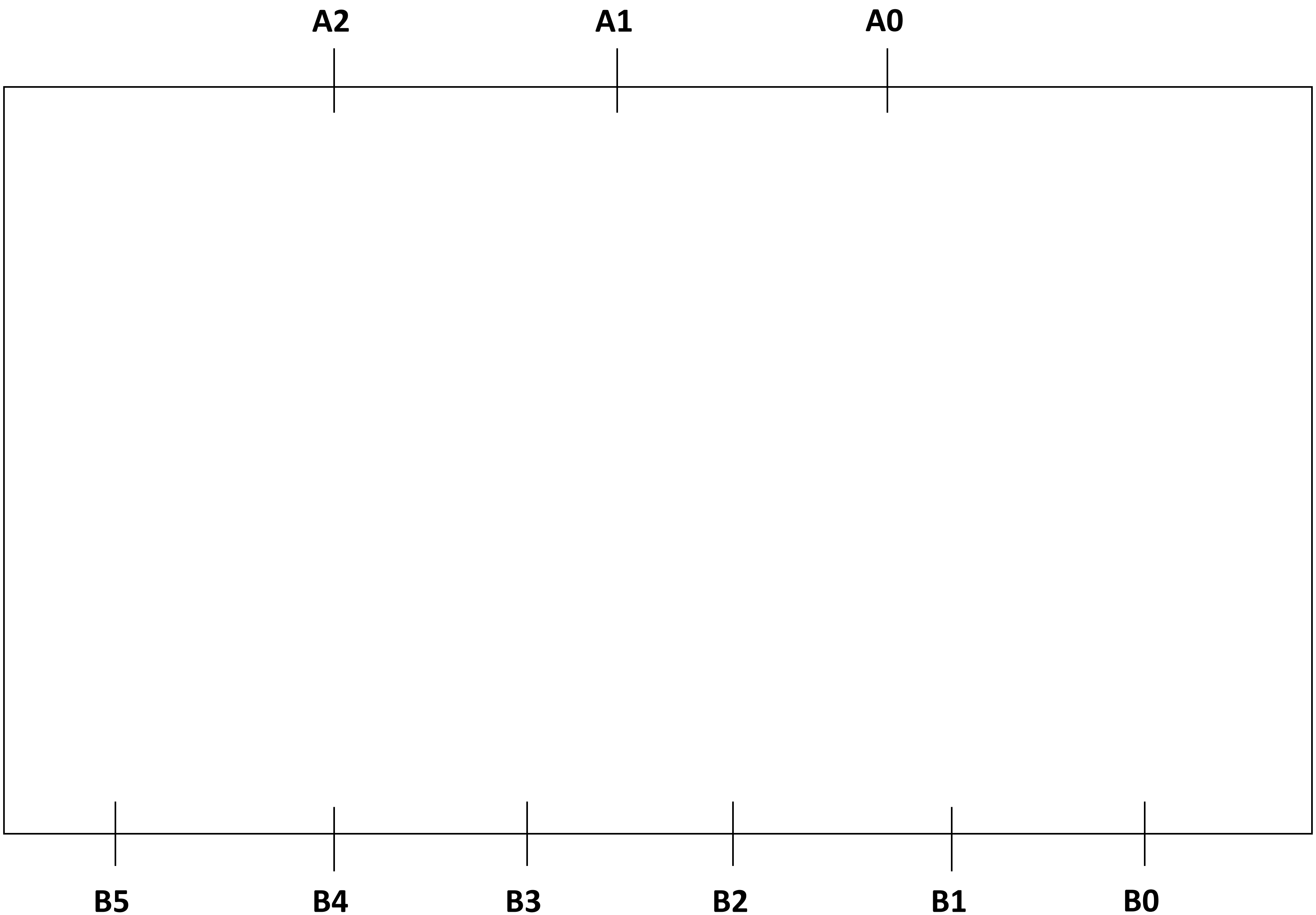
A0 \ A2A1	00	01	11	10
0	0	2	6	4
1	1	3	7	5

B4 =

A0 \ A2A1	00	01	11	10
0	0	2	6	4
1	1	3	7	5

B0 =

3-bit Squaring Circuit



If time permits…

FORMAL TERMINOLOGY FOR KMAPS

Terminology

- Implicant: A product term (grouping of 1's) that covers a subset of cases where F=1
  - the product term is said to "imply" F because if the product term evaluates to '1' then F='1'
- Prime Implicant: The largest grouping of 1's (smallest product term) that can be made
- Essential Prime Implicant: A prime implicant (product term) that is needed to cover all the 1's of F

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 0	(8) 0
01	(1) 1	(5) 1	(13) 0	(9) 0
11	(3) 1	(7) 1	(15) 1	(11) 1
10	(2) 0	(6) 0	(14) 1	(10) 1

An implicant

Not PRIME because not as large as possible

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 0	(8) 0
01	(1) 1	(5) 1	(13) 0	(9) 0
11	(3) 1	(7) 1	(15) 1	(11) 1
10	(2) 0	(6) 0	(14) 1	(10) 1

An implicant

Not PRIME because not as large as possible

An implicant

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	0: 0	4: 0	12: 0	8: 0
01	1: 1	5: 1	13: 0	9: 0
11	3: 1	7: 1	15: 1	11: 1
10	2: 0	6: 0	14: 1	10: 1

An implicant

Not PRIME because not as large as possible

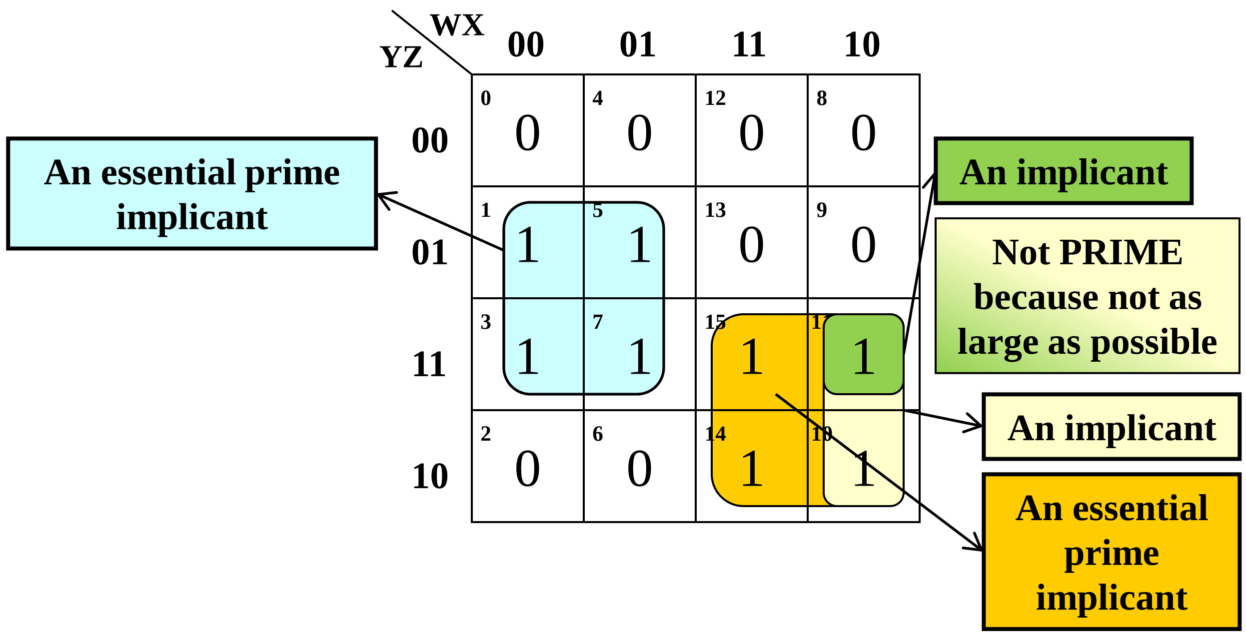
An implicant

An essential prime implicant

An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**

YZ \ WX	00	01	11	10
00	0 (0)	0 (4)	0 (12)	0 (8)
01	1 (1)	1 (5)	0 (13)	0 (9)
11	1 (3)	1 (7)	1 (15)	1 (11)
10	0 (2)	0 (6)	1 (14)	1 (10)

An essential prime implicant

A prime implicant, but not an ESSENTIAL implicant because it is not needed to cover all 1's in the function

An implicant

Not PRIME because not as large as possible

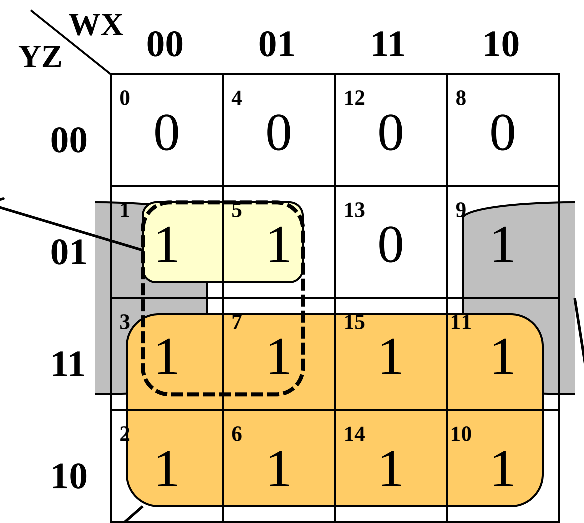
An implicant

An essential prime implicant

An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

Implicant Examples

W	X	Y	Z	F
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**1**
1	1	1	1	**1**



An implicant, but not a PRIME implicant because it is not as large as possible (should expand to combo's 3 and 7)

An essential prime implicant (largest grouping possible, that must be included to cover all 1's)

An essential prime implicant

K-Map Grouping Rules

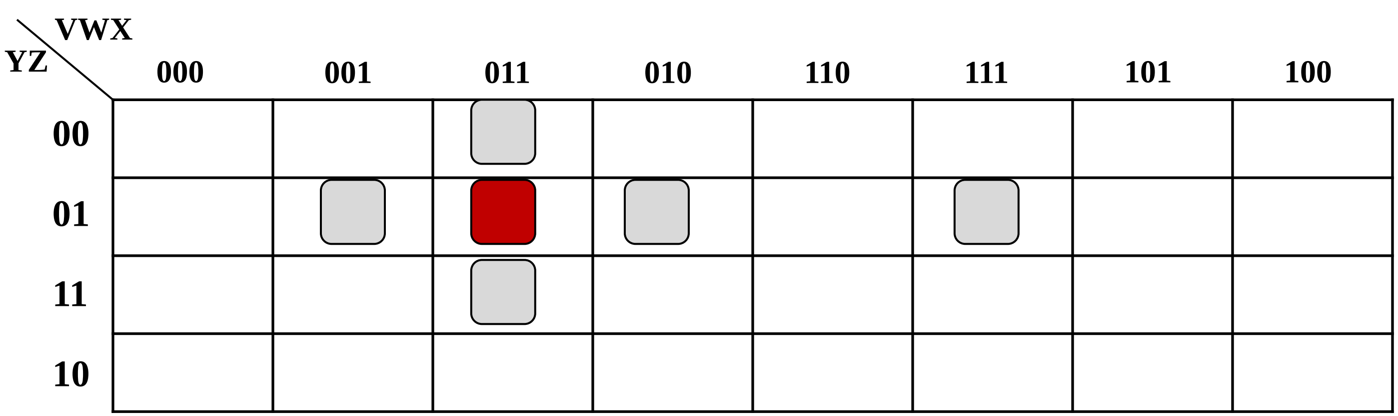
- Make groups (implicants) of 1, 2, 4, 8, ... and they must be rectangular or square in shape.
- Include the minimum number of essential prime implicants
  - Use only *essential* prime implicants (i.e. as few groups as possible to cover all 1's)
  - Ensure that you are using ***prime*** implicants (i.e. Always make groups as large as possible reusing squares if necessary)

Informational: You won't be asked to perform 5- or 6-variable K-Maps

5- & 6-VARIABLE KMAPS

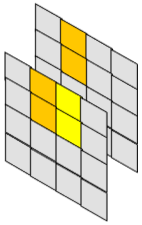
5-Variable K-Map

- If we have a 5-variable function we need a 32-square KMap.
- Will an 8x4 matrix work?
  - Recall K-maps work because adjacent squares differ by 1-bit
- How many adjacencies should we have for a given square?
- ___!! But drawn in 2 dimensions we can't have __ adjacencies.



5-Variable Karnaugh Maps

- To represent the 5 adjacencies of a 5-variable function [e.g. f(v,w,x,y,z)], imagine two 4x4 K-Maps stacked on top of each other
  - Adjacency across the two maps



YZ \ WX	00	01	11	10
00	0 0	4 1	12 1	8 0
01	1 0	5 1	13 1	9 0
11	3 0	7 0	15 0	11 0
10	2 0	6 0	14 0	10 0

V=0

These are adjacent

YZ \ WX	00	01	11	10
00	0 0	4 1	12 0	8 0
01	1 0	5 1	13 0	9 0
11	3 0	7 0	15 0	11 0
10	2 0	6 0	14 0	10 0

V=1

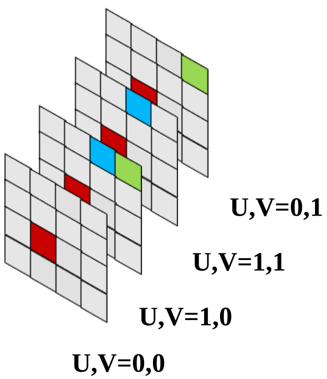
Traditional adjacencies still apply
(Note: v is constant for that group and should be included)
=> v'xy'

Adjacencies across the two maps apply
(Now v is not constant)
=> w'xy'

F = v'xy' + w'xy'

6-Variable Karnaugh Maps

- 6 adjacencies for 6-variables (Stack of four 4x4 maps)



YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 1	(8) 0
01	(1) 0	(5) 0	(13) 0	(9) 0
11	(3) 0	(7) 1	(15) 0	(11) 0
10	(2) 0	(6) 0	(14) 0	(10) 0

U,V=0,0

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 0	(8) 1
01	(1) 0	(5) 0	(13) 0	(9) 0
11	(3) 0	(7) 1	(15) 0	(11) 0
10	(2) 0	(6) 0	(14) 0	(10) 0

U,V=0,1

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 1	(8) 1
01	(1) 0	(5) 0	(13) 0	(9) 0
11	(3) 0	(7) 1	(15) 0	(11) 0
10	(2) 0	(6) 0	(14) 0	(10) 0

U,V=1,0

YZ \ WX	00	01	11	10
00	(0) 0	(4) 0	(12) 0	(8) 0
01	(1) 0	(5) 0	(13) 0	(9) 0
11	(3) 0	(7) 1	(15) 0	(11) 0
10	(2) 0	(6) 0	(14) 0	(10) 0

U,V=1,1

Group of 2

Not adjacent

Group of 4

7-Variable K-maps and Other Techniques

- Can we have 7-variable K-Maps?
- No! We would need to see 7 adjacencies per square and we humans cannot visualize 4 dimensions
- Other computer-friendly minimization algorithms
  - Quine-McCluskey
    - Still exponential runtime
    - Minimization is NP-hard problem
  - Espresso-heuristic Minimizer
    - Achieves "good" minimization in far less time (may not be absolute minimal)