# Unit 9

## Implementing Combinational Functions with Karnaugh Maps

# Outcomes

- I can use Karnaugh maps to synthesize combinational functions with several outputs
- I can determine the appropriate size and contents of a memory to implement any logic function (i.e. truth table)

# Covering Combinations

- A minterm corresponds to ("covers") 1 combination of a logic function
- As we remove variables from a product term, more combinations are covered
  - The product term will evaluate to true regardless of the removed variables value (i.e. the term is independent of that variable)

**F = WX'YZ**
**= m11**

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **0** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **0** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **0** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **0** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **0** |
| 1 | 0 | 1 | 0 | **0** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **0** |
| 1 | 1 | 1 | 1 | **0** |

**F = WX'Z**
**= m9+m11**

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **0** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **0** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **0** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **0** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **0** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **0** |
| 1 | 1 | 1 | 1 | **0** |

# Covering Combinations

- The more variables we can remove the more combinations a single product term covers
  - Said differently, a small term will cover (or expand to) more combinations
- The smaller the term, the smaller the circuit
  - We need fewer gates to check for multiple combinations
- For a given function, how can we find these smaller terms?

**F = X'Z**
**= m1+m3+m9+m11**

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **0** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **0** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **0** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **0** |
| 1 | 1 | 1 | 1 | **0** |

**F = X'**
**= m0+m1+m2+m3+m8+m9+m10+m11**

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **1** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **0** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **0** |
| 1 | 0 | 0 | 0 | **1** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **0** |
| 1 | 1 | 1 | 1 | **0** |

A new way to synthesize your logic functions

# KARNAUGH MAPS

# Logic Function Synthesis

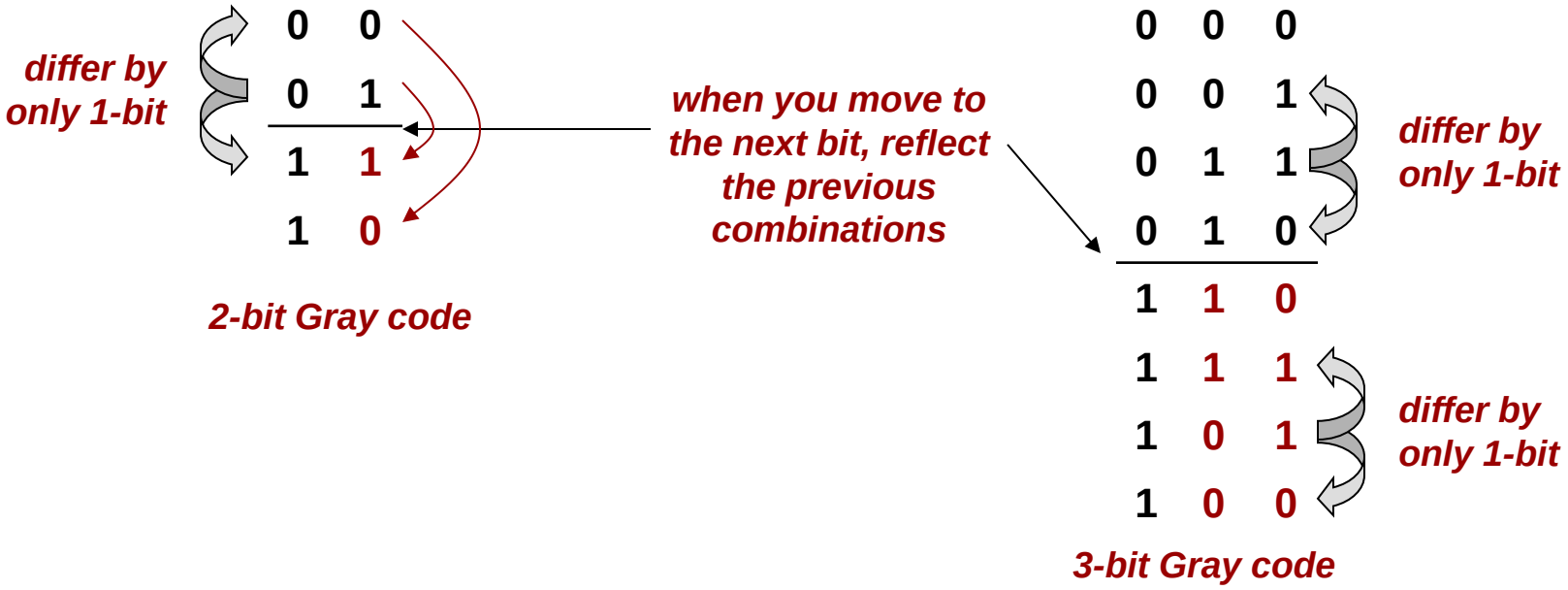
- Given a function description as a T.T. or sum of minterm (product of maxterm) form, how can we arrive at a circuit implementation or equation (i.e. perform logic synthesis)?
- Methods
  - Minterms / maxterms
    - Use Boolean Algebra to find minimal 2-level implementation
  - Karnaugh Maps [we will learn this one now]
    - Graphical method amenable to human visual inspection and can be used for functions of **up to 6 variables (but becomes large and unwieldy after just 4 variables)**
  - Quine-McCluskey Algorithm (amenable to computer implementations
  - Others: Espresso algorithm, Binary Decision Diagrams, etc.

# Karnaugh Maps

- If used correctly, will always yield a minimal, **2-level implementation**
  - There may be a more minimal 3-level, 4-level, 5-level… implementation but K-maps produce the minimal two-level (SOP or POS) implementation
- Represent the truth table graphically as a series of adjacent squares that allows a human to see where variables can be removed

# Gray Code

- Different than normal binary ordering
- Reflective code
  - When you add the $(n+1)^{th}$ bit, reflect all the previous n-bit combinations
- Consecutive code words differ by only 1-bit

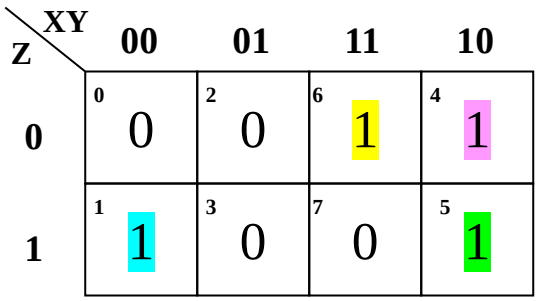


# Karnaugh Map Construction

- Every square represents 1 input combination
- Must label axes in Gray code order
- Fill in squares with given function values

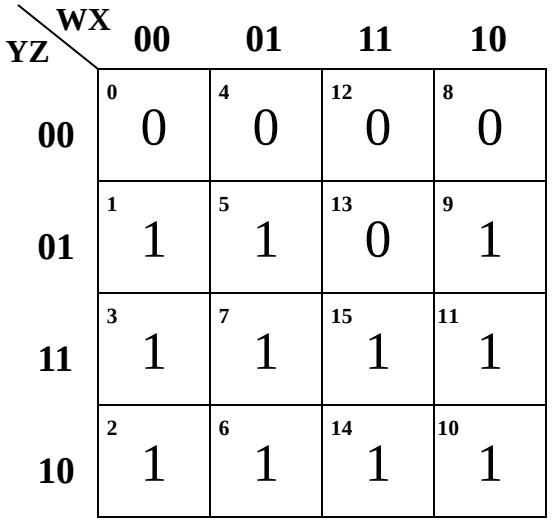
| X | Y | Z | F |
|---|---|---|---|
| 0 | 0 | 0 | **0** |
| 0 | 0 | 1 | **1** |
| 0 | 1 | 0 | **0** |
| 0 | 1 | 1 | **0** |
| 1 | 0 | 0 | **1** |
| 1 | 0 | 1 | **1** |
| 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | **0** |

**F(x,y,z)=m1 + m4 + m5 + m6**




**3 Variable Karnaugh Map**

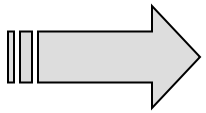
**G(w,x,y,z)=m1+m2+m3+m5+m6+m7+m9+ m10+m11+m14+m15**




**4 Variable Karnaugh Map**

# Karnaugh Maps

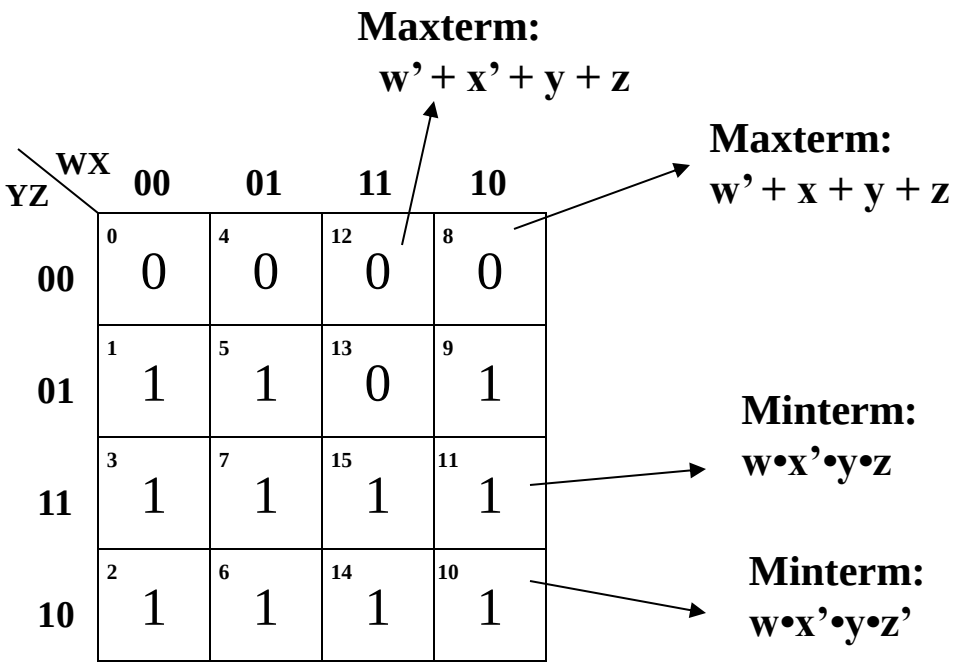
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |



| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **00** | 0: 0 | 4: 0 | 12: 0 | 8: 0 |
| **01** | 1: 1 | 5: 1 | 13: 0 | 9: 1 |
| **11** | 3: 1 | 7: 1 | 15: 1 | 11: 1 |
| **10** | 2: 1 | 6: 1 | 14: 1 | 10: 1 |

# Karnaugh Maps

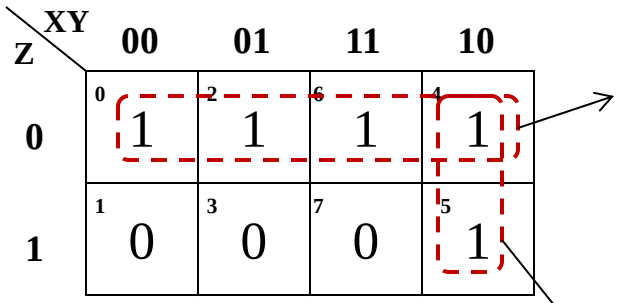
- Squares with a '1' represent minterms that must be included in the SOP solution
- Squares with a '0' represent maxterms that must be included in the POS solution



# Karnaugh Maps

- Groups (of 2, 4, 8, etc.) of adjacent 1's will always simplify to smaller product term than just individual minterms

**F=m0+m2+m4+m5+m6**




**3 Variable Karnaugh Map**

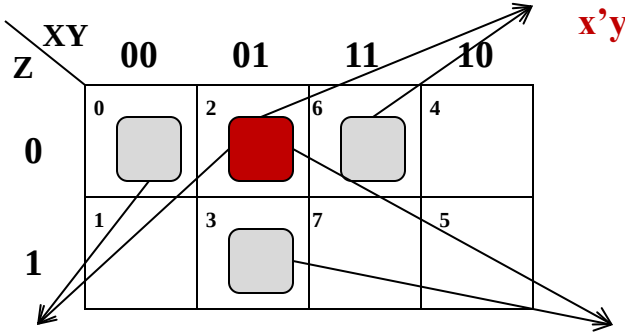
**= m0 + m2 + m6 + m4**
**= x'y'z' + x'yz' + xyz' + xy'z'**
**= z'(x'y' + x'y + xy + xy')**
**= z'(x'(y'+y) + x(y+y'))**
**= z'(x'+x)**
**= z'**

**= m4 + m5**
**= xy'z' + xy'z = xy'(z'+z)**
**= xy'**

# Karnaugh Maps

- Adjacent squares differ by 1-variable
  - This will allow us to use T10 = AB + AB'= A or T10' = (A+B')(A+B) = A

**3 Variable Karnaugh Map**




**Difference in X: 010 & 110**
**x'yz' + xyz' = yz'**

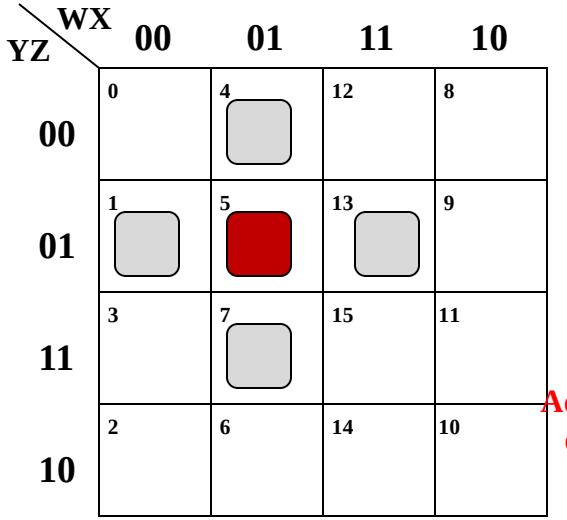
**Difference in Y: 010 & 000**
**x'yz' + x'y'z' = x'z'**

**Difference in Z: 010 & 011**
**x'yz' + x'yz = x'y**

**0 = 0<u>0</u>0**
**2 = <u>010</u>**
**3 = 01<u>1</u>**
**6 = <u>1</u>10**

**Adjacent squares differ by 1-bit**

**4 Variable Karnaugh Map**




**1 = 0<u>0</u>01**
**4 = 010<u>0</u>**
**5 = <u>0101</u>**
**7 = 01<u>1</u>1**
**13 = <u>1</u>101**

**Adjacent squares differ by 1-bit**

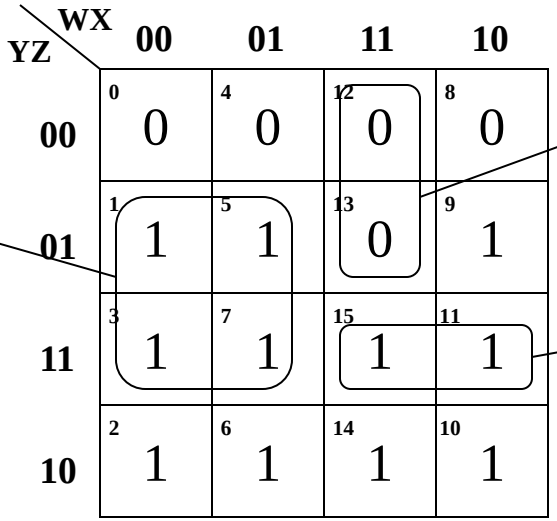
# Karnaugh Maps

- 2 adjacent 1's (or 0's) differ by only one variable
- 4 adjacent 1's (or 0's) differ by two variables
- 8, 16, … adjacent 1's (or 0's) differ by 3, 4, … variables
- By grouping adjacent squares with 1's (or 0's) in them, we can come up with a simplified expression using T10 (or T10' for 0's)

w'•x'•y'•z + w'•x'•y•z +
w'•x•y'•z + w'•x•y•z

= w'•z

*w'z are constant while all combos of x and y are present (x'y', x'y, xy', xy)*



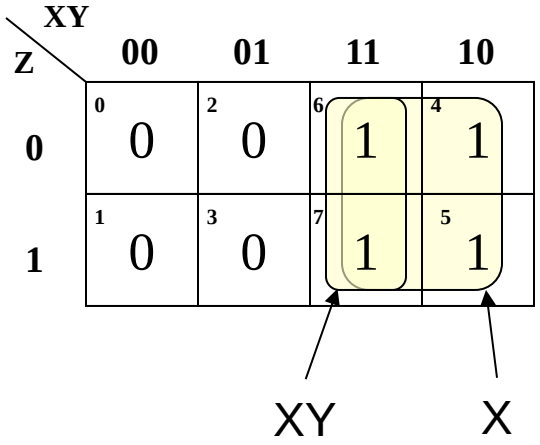

(w'+x'+y+z)•(w'+x'+y+z') =
(w'+x'+y)

w•x•y•z + w•x'•y•z =
w•y•z

# K-Map View of the Theorems

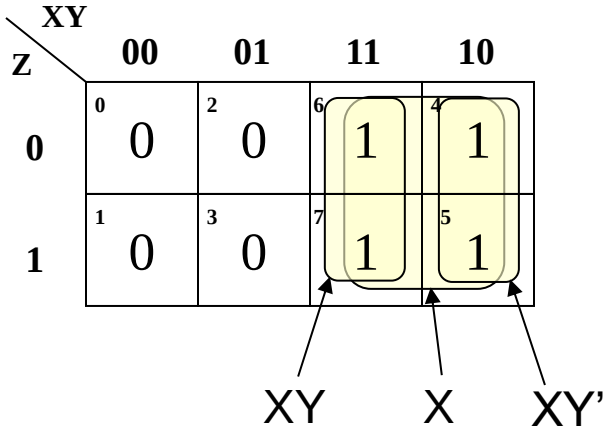
- The 2 & 3 variable theorems used to simplify expressions can be illustrated using K-Maps.

## T9: Covering
$X + XY = X$



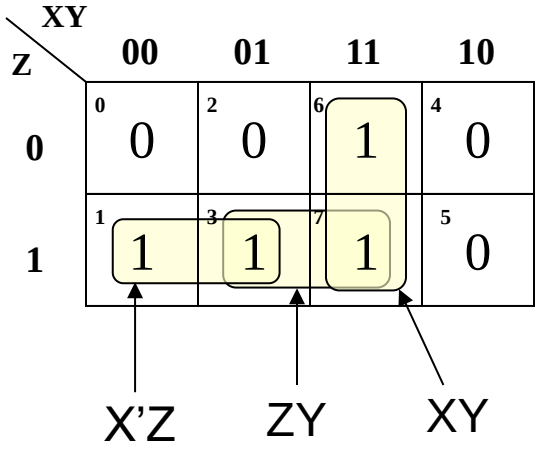

X "covers" XY so XY not needed

## T10: Combining
$XY + XY' = X$




XY and XY' can be combined to form X

## T11: Consensus
$XY + X'Z + ZY = XY + X'Z$




Don't need ZY if you have X'Z and XY

# K-Map Grouping Rules

- Cover the 1's [=on-set] or 0's [=off-set] with as few groups as possible, but make those groups as large as possible
  - Make them as large as possible even if it means "covering" a 1 (or 0) that's already a member of another group
- Make groups of 1, 2, 4, 8, ... and they must be rectangular or square in shape.
- Wrapping is legal

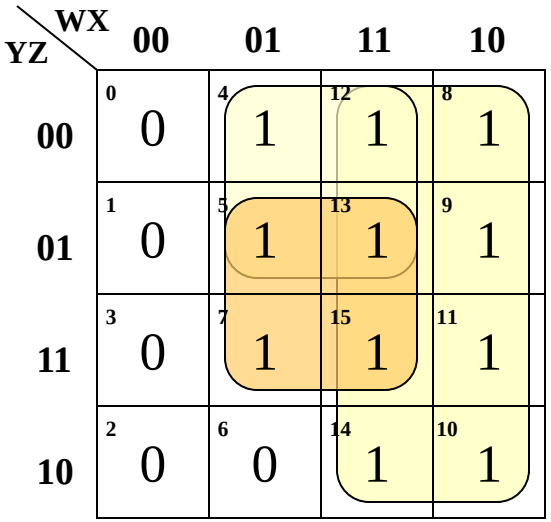
# Group These K-Maps

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0: 0 | 2: 1 | 6: 0 | 4: 0 |
| 1 | 1: 1 | 3: 0 | 7: 0 | 5: 0 |

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0: 1 | 2: 1 | 6: 0 | 4: 0 |
| 1 | 1: 1 | 3: 0 | 7: 0 | 5: 0 |

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0: 0 | 4: 0 | 12: 1 | 8: 1 |
| 01 | 1: 1 | 5: 1 | 13: 1 | 9: 0 |
| 11 | 3: 1 | 7: 1 | 15: 1 | 11: 0 |
| 10 | 2: 0 | 6: 0 | 14: 0 | 10: 1 |

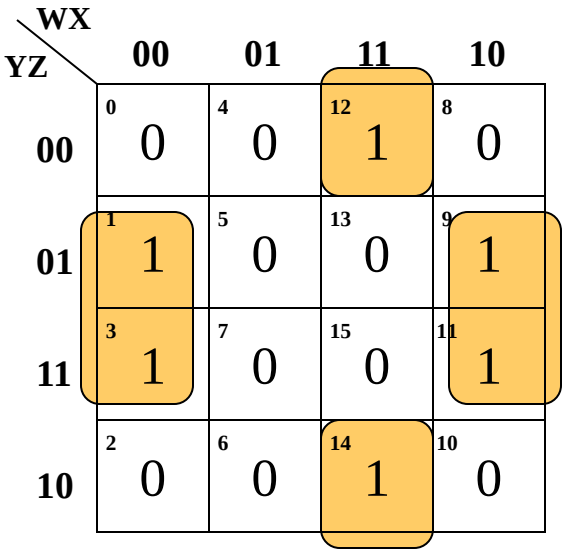
# Karnaugh Maps



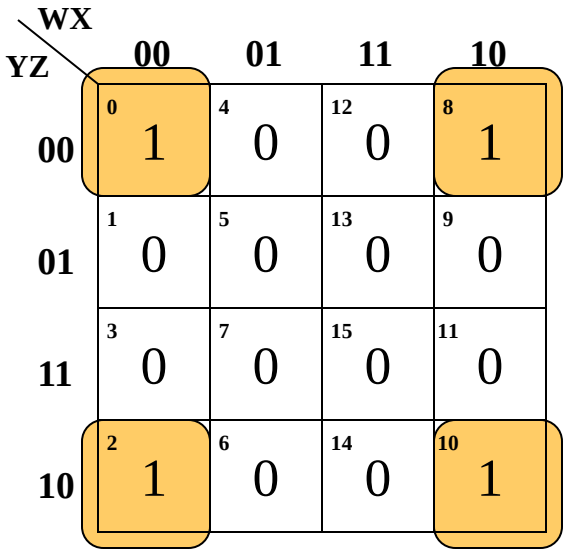

- Cover the remaining ‘1’ with the largest group possible even if it “reuses” already covered 1’s

# Karnaugh Maps

- Groups can wrap around from:
  - Right to left
  - Top to bottom
  - Corners




F = X'Z + WXZ'




F = X'Z'

# Group This

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0: 0 | 4: 0 | 12: 0 | 8: 0 |
| 01 | 1: 1 | 5: 1 | 13: 0 | 9: 1 |
| 11 | 3: 1 | 7: 1 | 15: 1 | 11: 1 |
| 10 | 2: 1 | 6: 1 | 14: 1 | 10: 1 |

# K-Map Translation Rules

- When translating a group of 1's, find the variable values that are constant for each square in the group and translate only those variables values to a product term
- Grouping 1's yields SOP
- When translating a group of 0's, again find the variable values that are constant for each square in the group and translate only those variable values to a sum term
- Grouping 0's yields POS

# Karnaugh Maps (SOP)

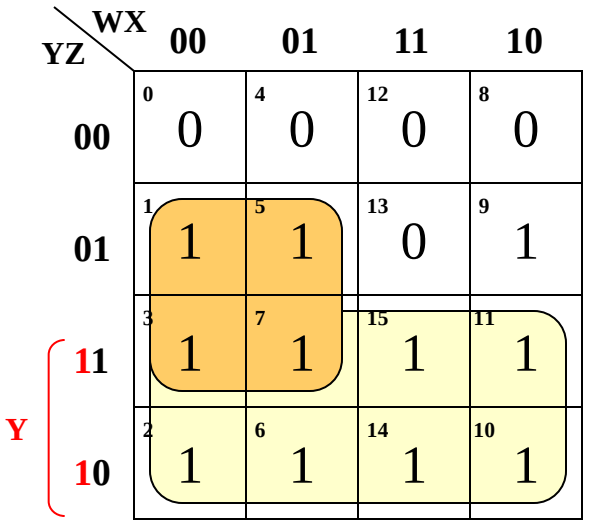
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **00** | 0 (0) | 0 (4) | 0 (12) | 0 (8) |
| **01** | 1 (1) | 1 (5) | 0 (13) | 1 (9) |
| **11** | 1 (3) | 1 (7) | 1 (15) | 1 (11) |
| **10** | 1 (2) | 1 (6) | 1 (14) | 1 (10) |

**F =**

# Karnaugh Maps (SOP)

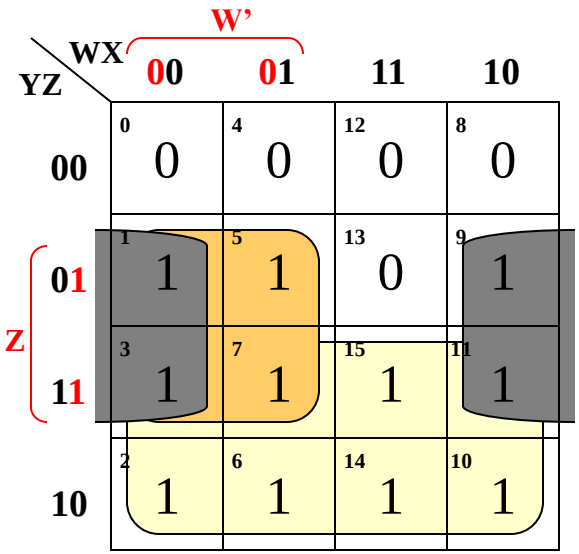
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |




**F = Y**

# Karnaugh Maps (SOP)

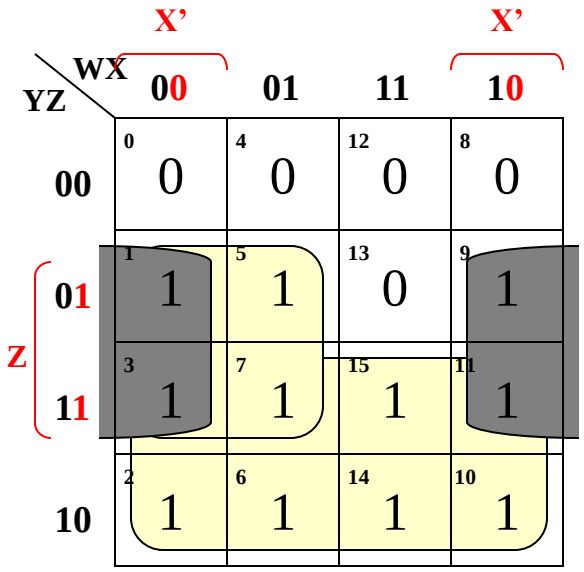
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |




**F = Y + W'Z + …**

# Karnaugh Maps (SOP)

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |




$F = Y + W'Z + X'Z$

# Karnaugh Maps (POS)

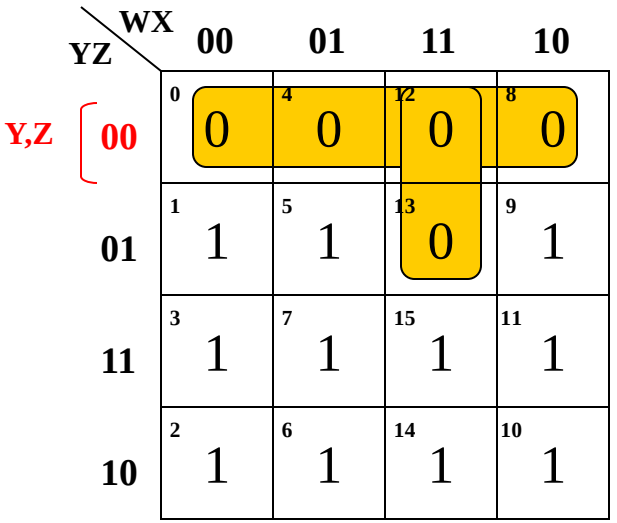
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 (0) | 4 0 | 12 0 | 8 0 |
| 01 | 1 1 | 5 1 | 13 0 | 9 1 |
| 11 | 3 1 | 7 1 | 15 1 | 11 1 |
| 10 | 2 1 | 6 1 | 14 1 | 10 1 |

**F =**

# Karnaugh Maps (POS)

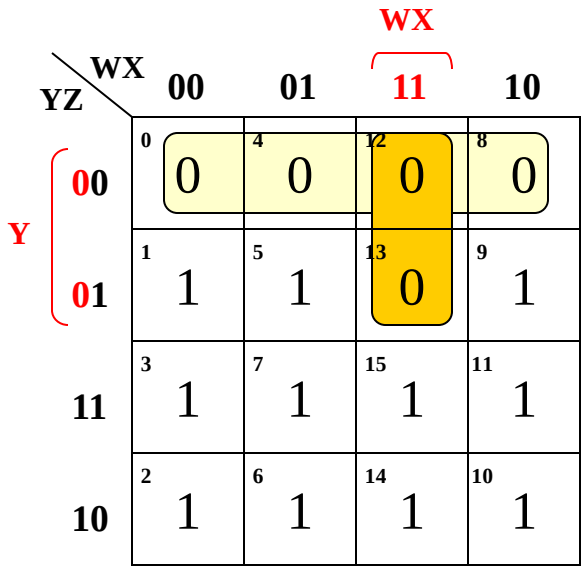
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |




**F = (Y+Z)**

# Karnaugh Maps (POS)

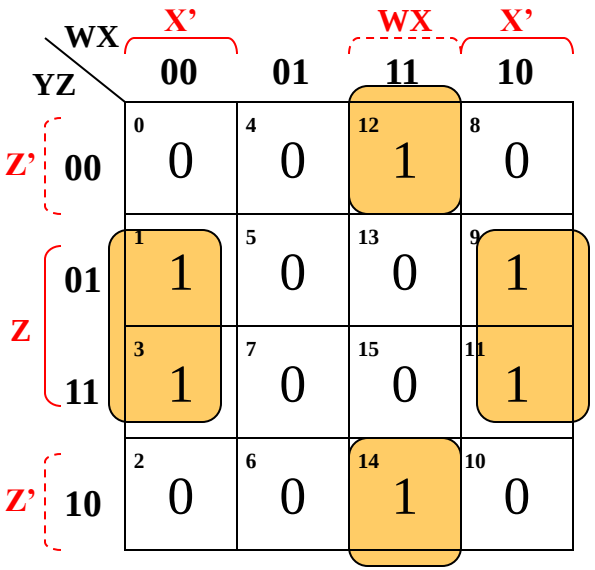
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |



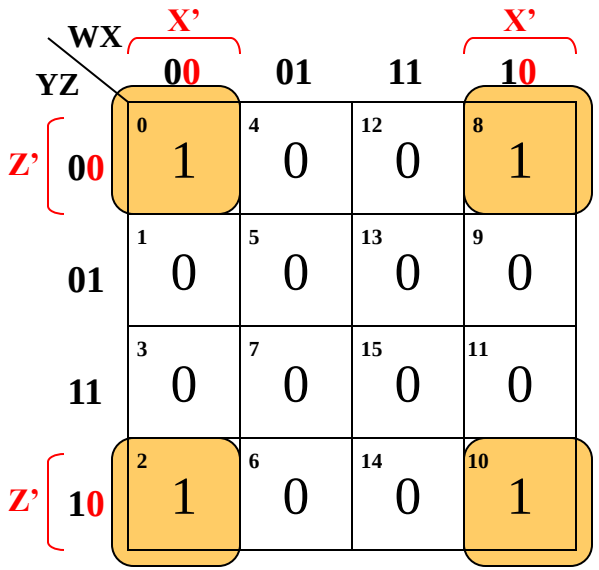

**F = (Y+Z)(W'+X'+Y)**

# Karnaugh Maps

- Groups can wrap around from:
  - Right to left
  - Top to bottom
  - Corners




F = X'Z + WXZ'




F = X'Z'

# Exercises

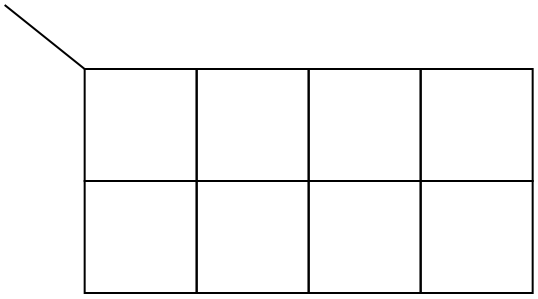
| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0: 1 | 4: 0 | 12: 0 | 8: 1 |
| 01 | 1: 1 | 5: 0 | 13: 0 | 9: 1 |
| 11 | 3: 0 | 7: 0 | 15: 0 | 11: 0 |
| 10 | 2: 1 | 6: 0 | 14: 1 | 10: 1 |

$F_{SOP}$=

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0: 1 | 4: 0 | 12: 0 | 8: 1 |
| 01 | 1: 1 | 5: 0 | 13: 0 | 9: 1 |
| 11 | 3: 0 | 7: 0 | 15: 0 | 11: 0 |
| 10 | 2: 1 | 6: 0 | 14: 1 | 10: 1 |

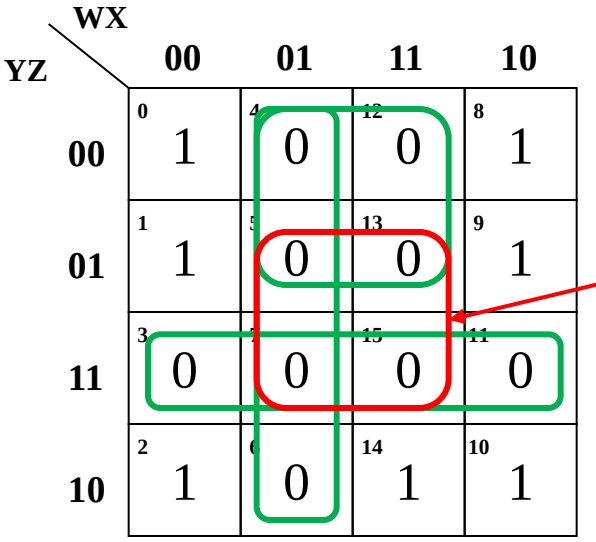
$F_{POS}$=

P(x,y,z)=m2+m3+m5+m7



P=

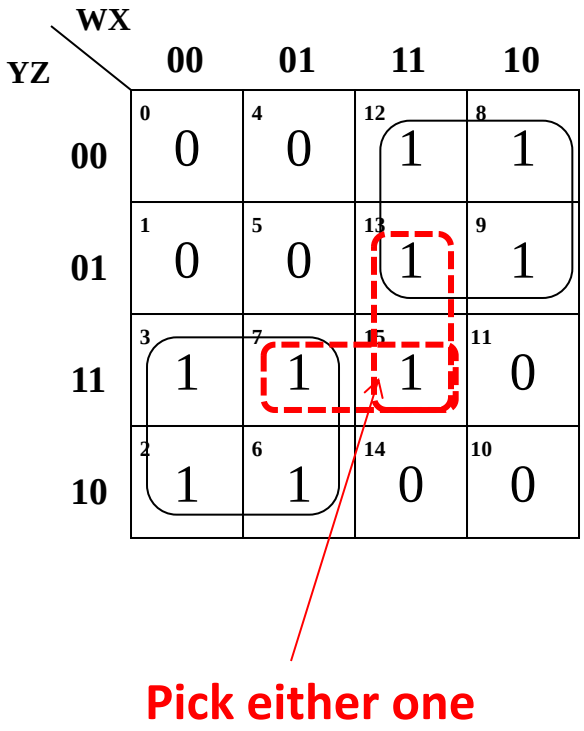
# No Redundant Groups




**This group does not cover new squares that are not already part of another essential grouping**

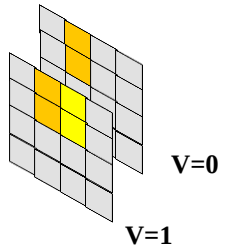
# Multiple Minimal Expressions

- For some functions, multiple minimal expressions (multiple minimal groups) exist…Pick one

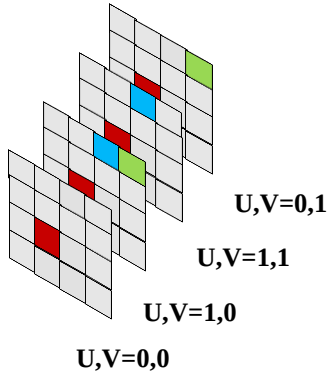


# Karnaugh Maps Beyond 4 Variables

- Recall, K-Maps require an adjacency for each variable
  - To see the necessary adjacencies, 5 and 6 variable K-Maps can be thought of in three dimensions
- Can we have 7-variable K-Maps?
  - No! We would need to see 7 adjacencies per square and we humans cannot visualize 4 dimensions
- Other computer-friendly minimization algorithms
  - Quine-McCluskey
    - Still exponential runtime
    - Minimization is NP-hard problem
  - Espresso-heuristic Minimizer
    - Achieves "good" minimization in far less time (may not be absolute minimal)




**5 Variable K-Maps**




**6 Variable K-Maps**

# DON'T CARE OUTPUTS

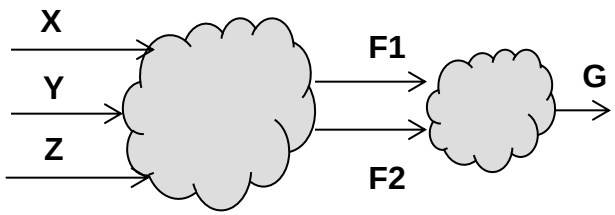
# Don't-Cares

- Sometimes there are certain input combinations that are illegal (due to physical or other external constraints)
- The outputs for the illegal inputs are "don't-cares"
  - The output can either be 0 or 1 since the inputs can never occur
  - Don't-cares can be included in groups of 1 or groups of 0 when grouping in K-Maps
  - Use them to make as big of groups as possible

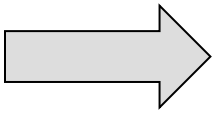
**Use 'Don't care' outputs as wildcards (e.g. the blank tile in Scrabble™). They can be either 0 or 1 whatever helps make bigger groups to cover the ACTUAL 1's**

# Combining Functions

- Given intermediate functions F1 and F2, how could you use AND, OR, NOT to make G
- Notice certain F1,F2 combinations never occur in G(x,y,z)…what should we make their output in the T.T.



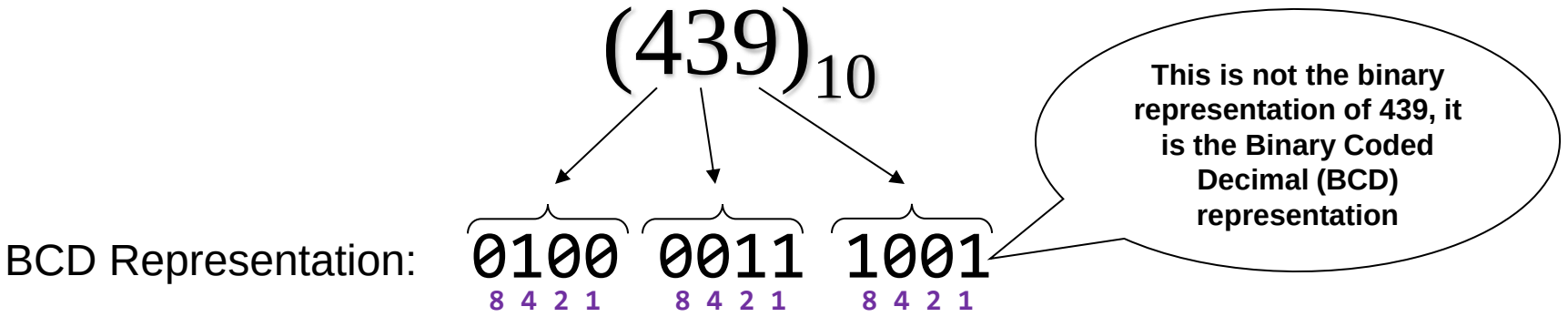

| X | Y | Z | F1 | F2 | G |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |



| F1 | F2 | G |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | |
| 1 | 0 | |
| 1 | 1 | |

# Invalid Input Combinations

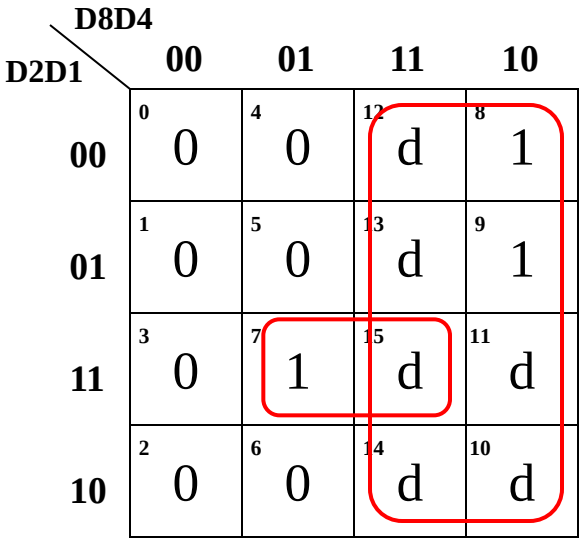
- An example of where Don't-Cares may come into play is Binary Coded Decimal (BCD)
  - Rather than convert a decimal number to unsigned binary (i.e. summing increasing powers of 2) we can represent each decimal digit as a separate group of 4-bits (with weights 8,4,2,1 for each group of 4 bits)
  - **Combinations 1010-1111 cannot occur!**



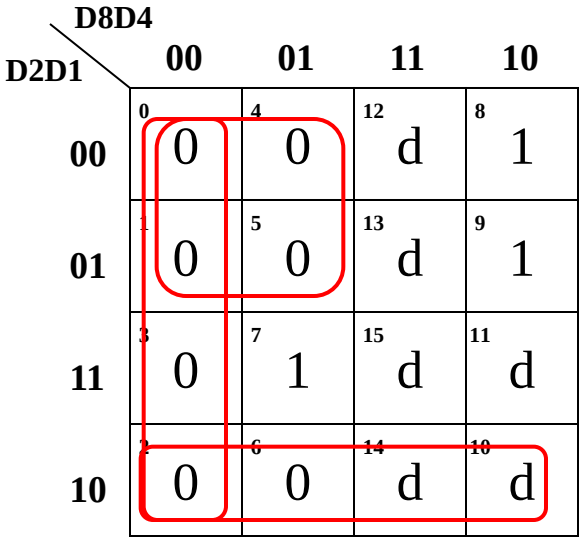

**Important: BCD represent each decimal digit with a separate group of bits**

# Don't Care Example

| D8 | D4 | D2 | D1 | GT6 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **0** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **0** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **0** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **1** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **d** |
| 1 | 0 | 1 | 1 | **d** |
| 1 | 1 | 0 | 0 | **d** |
| 1 | 1 | 0 | 1 | **d** |
| 1 | 1 | 1 | 0 | **d** |
| 1 | 1 | 1 | 1 | **d** |



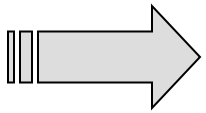

$GT6_{SOP}=$

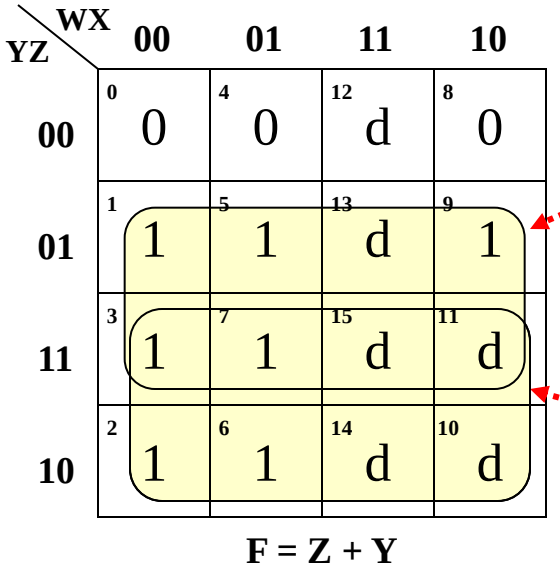



$GT6_{POS}=$

# Don't Cares

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **d** |
| 1 | 0 | 1 | 1 | **d** |
| 1 | 1 | 0 | 0 | **d** |
| 1 | 1 | 0 | 1 | **d** |
| 1 | 1 | 1 | 0 | **d** |
| 1 | 1 | 1 | 1 | **d** |





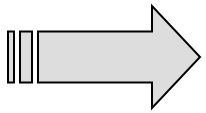

F = Z + Y

Reuse "d's" to make as large a group as possible to cover 1,5, & 9

Use these 4 "d's" to make a group of 8

# Don't Cares

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **d** |
| 1 | 0 | 1 | 1 | **d** |
| 1 | 1 | 0 | 0 | **d** |
| 1 | 1 | 0 | 1 | **d** |
| 1 | 1 | 1 | 0 | **d** |
| 1 | 1 | 1 | 1 | **d** |



| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 (0) | 4: 0 | 12: d | 8: 0 |
| 01 | 1: 1 | 5: 1 | 13: d | 9: 1 |
| 11 | 3: 1 | 7: 1 | 15: d | 11: d |
| 10 | 2: 1 | 6: 1 | 14: d | 10: d |

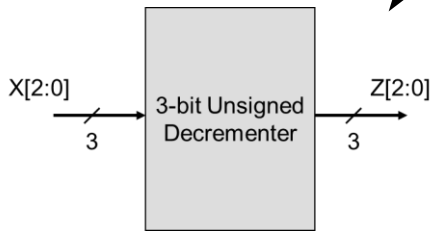
**F = Y+Z**

**You can use "d's" when grouping 0's and converting to POS**

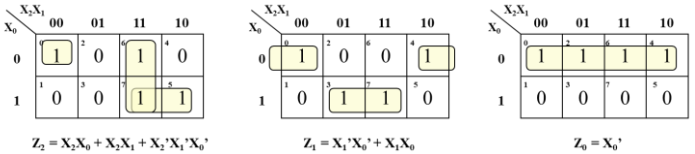
# A GENERAL, COMBINATIONAL CIRCUIT DESIGN PROCESS

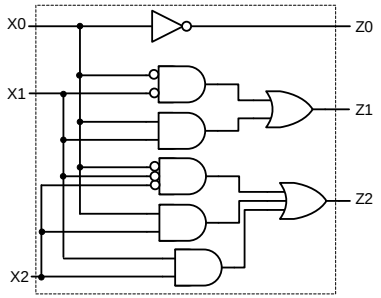
# Combinational Design Process

- Understand the problem
  - How many input bits and their representation system
  - How many output bits need be generated and what are their representation
  - Draw a block diagram
- Write a truth table
- Use a K-map to derive an equation for EACH output bit
- Use the equation to draw a circuit for EACH output bit, letting each circuit run in parallel to produce their respective output bit



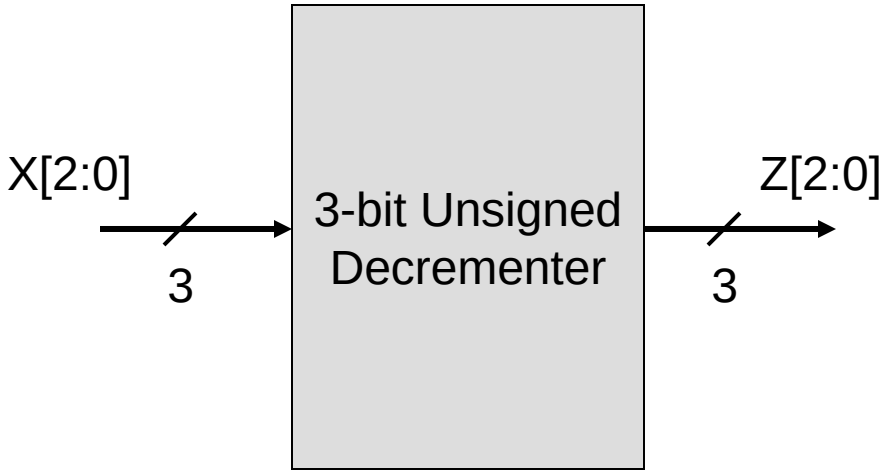

| $X_2$ | $X_1$ | $X_0$ | $Z_2$ | $Z_1$ | $Z_0$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | **1** | **1** | **1** |
| 0 | 0 | 1 | **0** | **0** | **0** |
| 0 | 1 | 0 | **0** | **0** | **1** |
| 0 | 1 | 1 | **0** | **1** | **0** |
| 1 | 0 | 0 | **0** | **1** | **1** |
| 1 | 0 | 1 | **1** | **0** | **0** |
| 1 | 1 | 0 | **1** | **0** | **1** |
| 1 | 1 | 1 | **1** | **1** | **0** |



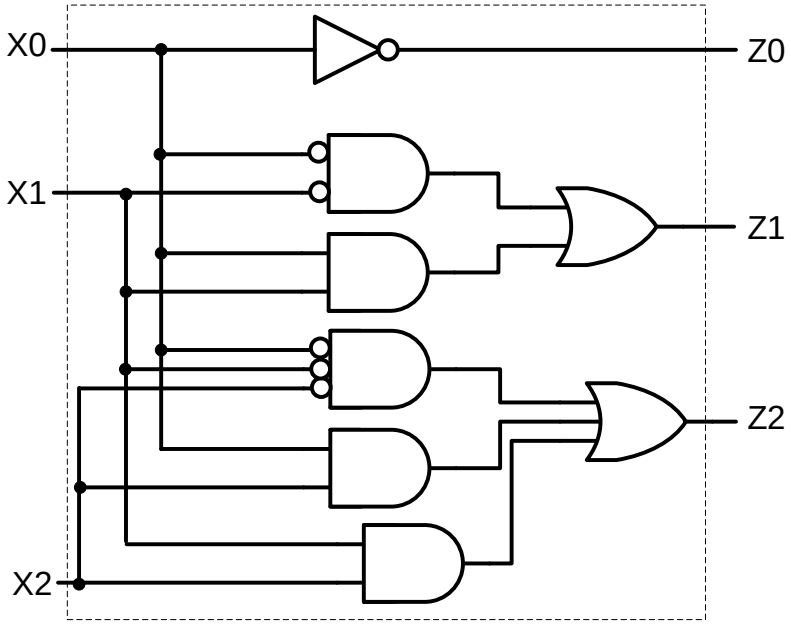
# Designing Circuits w/ K-Maps

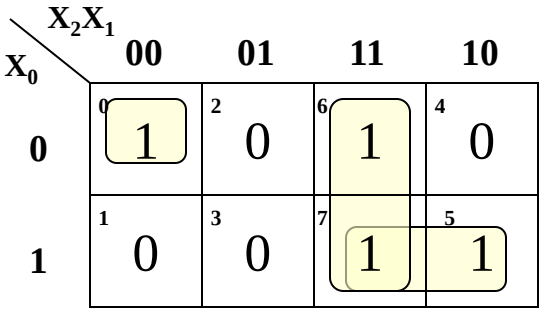
- Given a description…
  - Block Diagram
  - Truth Table
  - K-Map for each output bit (each output bit is a separate function of the inputs)
- 3-bit unsigned decrementer (Z = X-1)
  - If X[2:0] = 000 then Z[2:0] = 111, etc.



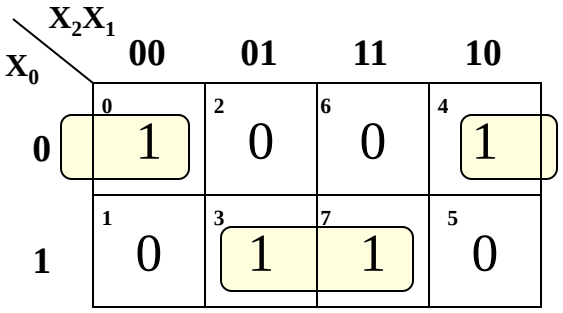
# 3-bit Number Decrementer

| $X_2$ | $X_1$ | $X_0$ | $Z_2$ | $Z_1$ | $Z_0$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | **1** | **1** | **1** |
| 0 | 0 | 1 | **0** | **0** | **0** |
| 0 | 1 | 0 | **0** | **0** | **1** |
| 0 | 1 | 1 | **0** | **1** | **0** |
| 1 | 0 | 0 | **0** | **1** | **1** |
| 1 | 0 | 1 | **1** | **0** | **0** |
| 1 | 1 | 0 | **1** | **0** | **1** |
| 1 | 1 | 1 | **1** | **1** | **0** |

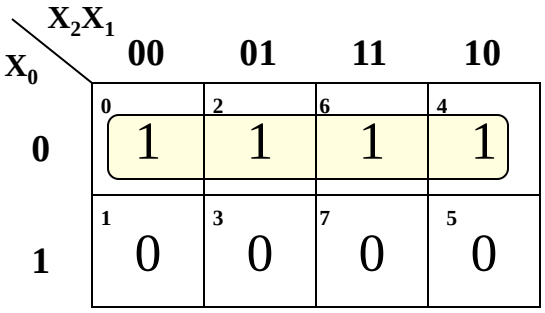



$Z_2 = X_2X_0 + X_2X_1 + X_2'X_1'X_0'$




$Z_1 = X_1'X_0' + X_1X_0$



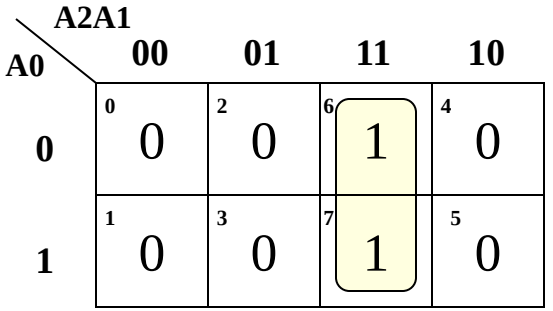

$Z_0 = X_0'$

# Squaring Circuit

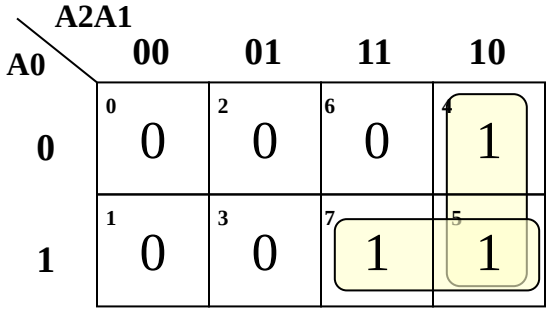
- Design a combinational circuit that accepts a 3-bit number and generates an output binary number equal to the square of the input number. ($B = A^2$)
- Using 3 bits we can represent the numbers from _____ to _____ .
- The possible squared values range from _____ to _____ .
- Thus to represent the possible outputs we need how many bits? _____

# 3-bit Squaring Circuit

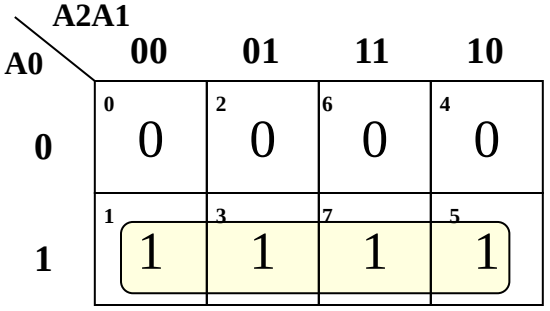
| A | Inputs $A_2$ | $A_1$ | $A_0$ | Outputs $B_5$ | $B_4$ | $B_3$ | $B_2$ | $B_1$ | $B_0$ | $B=A^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 4 |
| 3 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 9 |
| 4 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 16 |
| 5 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 25 |
| 6 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 36 |
| 7 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 49 |




**B5 = A2A1**

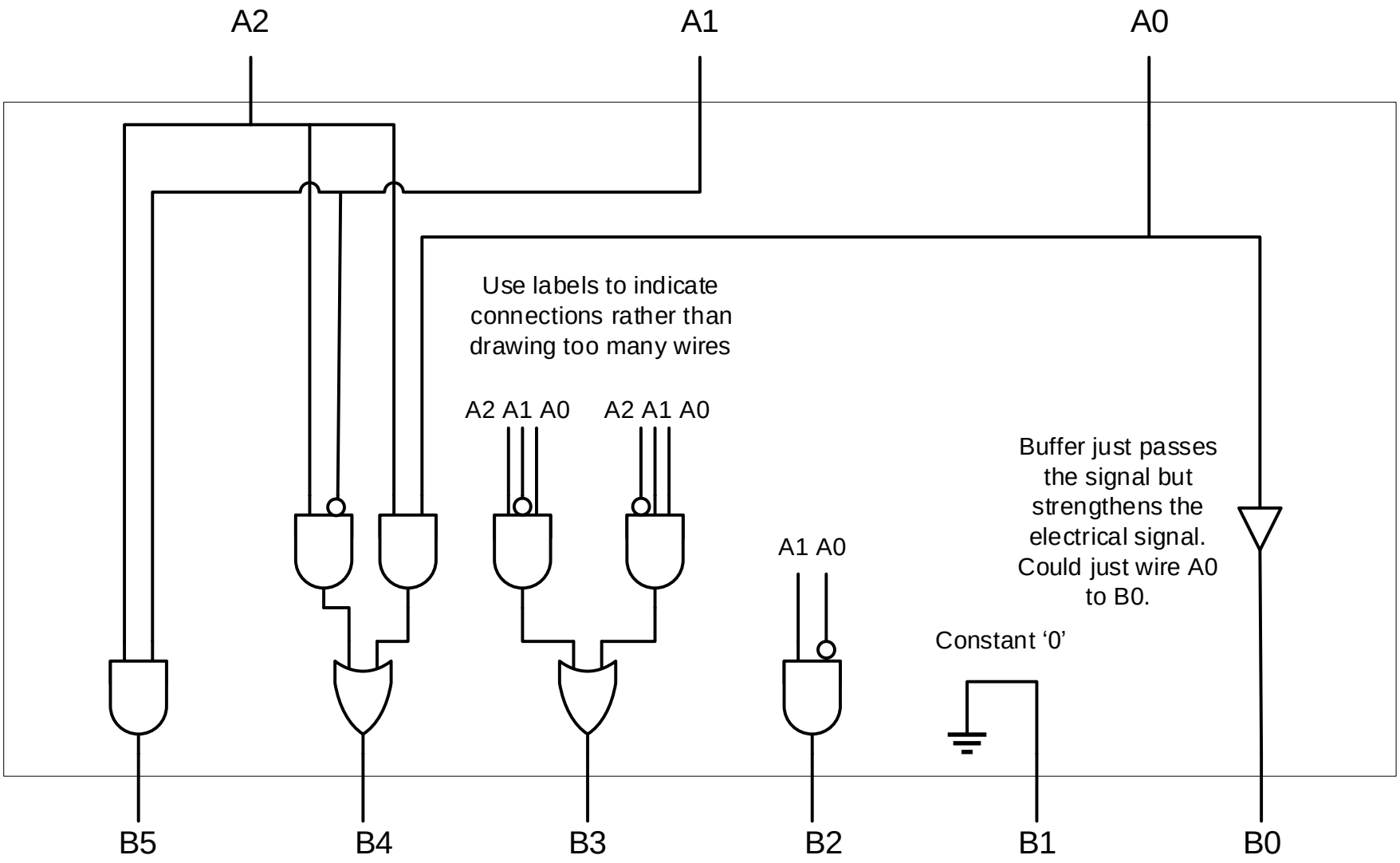



**B4 = A2A0 + A2A1'**




**B0 = A0**

# 3-bit Squaring Circuit Sol



If time permits…

# FORMAL TERMINOLOGY FOR KMAPS

# Terminology

- Implicant: A product term (grouping of 1's) that covers a subset of cases where F=1
  - the product term is said to "imply" F because if the product term evaluates to '1' then F='1'
- Prime Implicant: The largest grouping of 1's (smallest product term) that can be made
- Essential Prime Implicant: A prime implicant (product term) that is needed to cover all the 1's of F

# Implicant Examples

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **0** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |

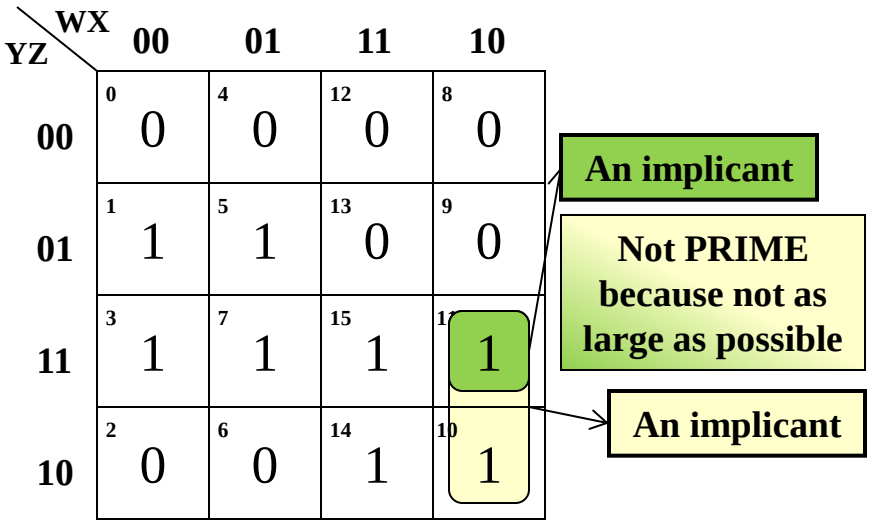
| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0: 0 | 4: 0 | 12: 0 | 8: 0 |
| 01 | 1: 1 | 5: 1 | 13: 0 | 9: 0 |
| 11 | 3: 1 | 7: 1 | 15: 1 | 1[illegible]: 1 |
| 10 | 2: 0 | 6: 0 | 14: 1 | 10: 1 |

**An implicant**

**Not PRIME because not as large as possible**

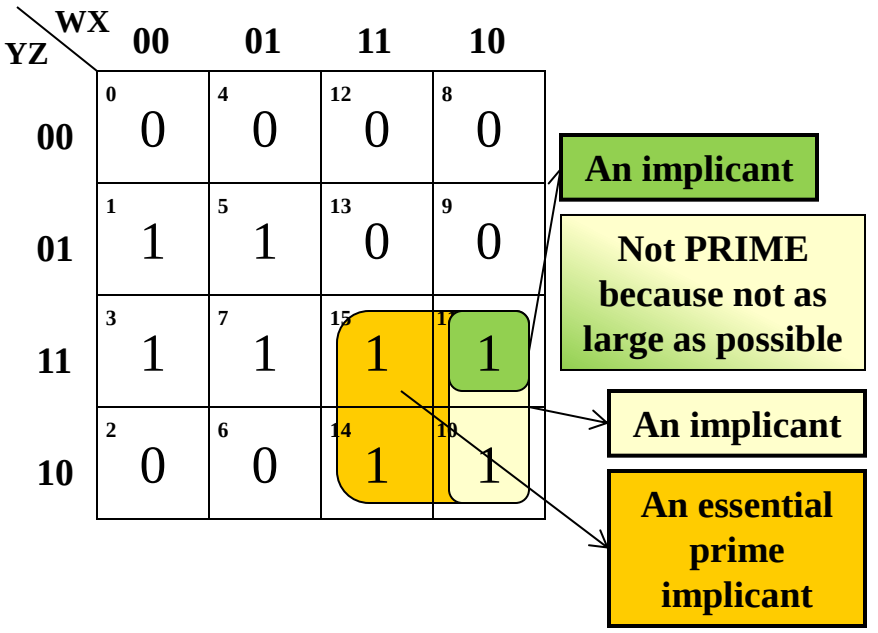
# Implicant Examples

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **0** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |



# Implicant Examples

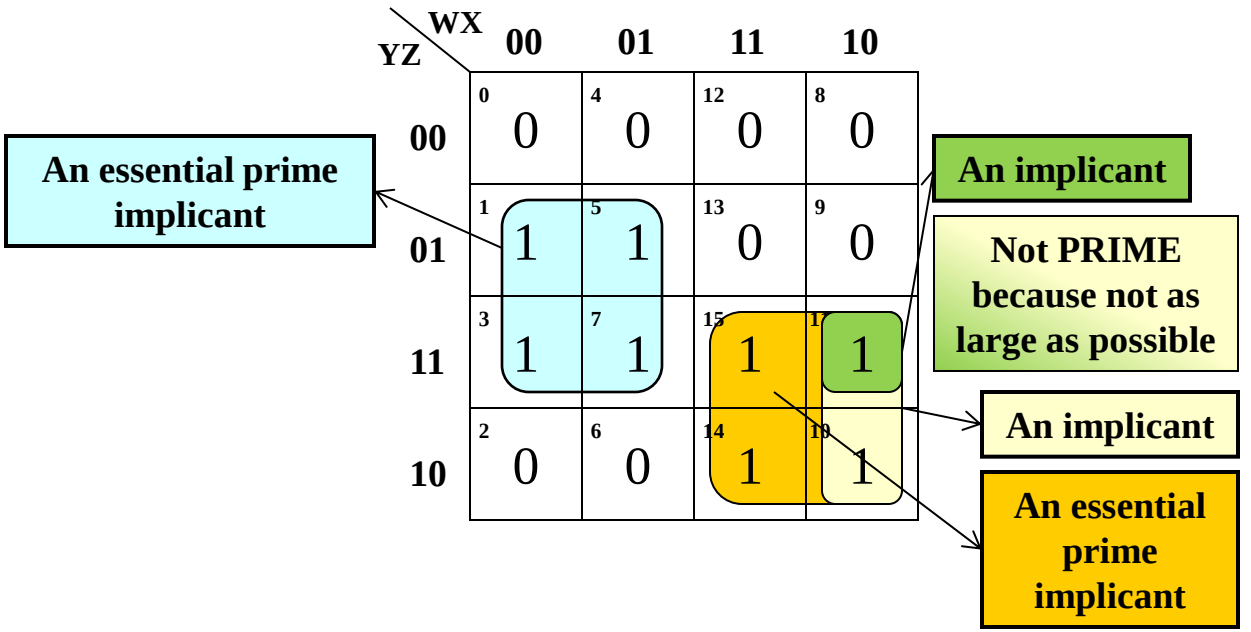
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **0** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |




**An essential prime implicant (largest grouping possible, that must be included to cover all 1's)**

# Implicant Examples

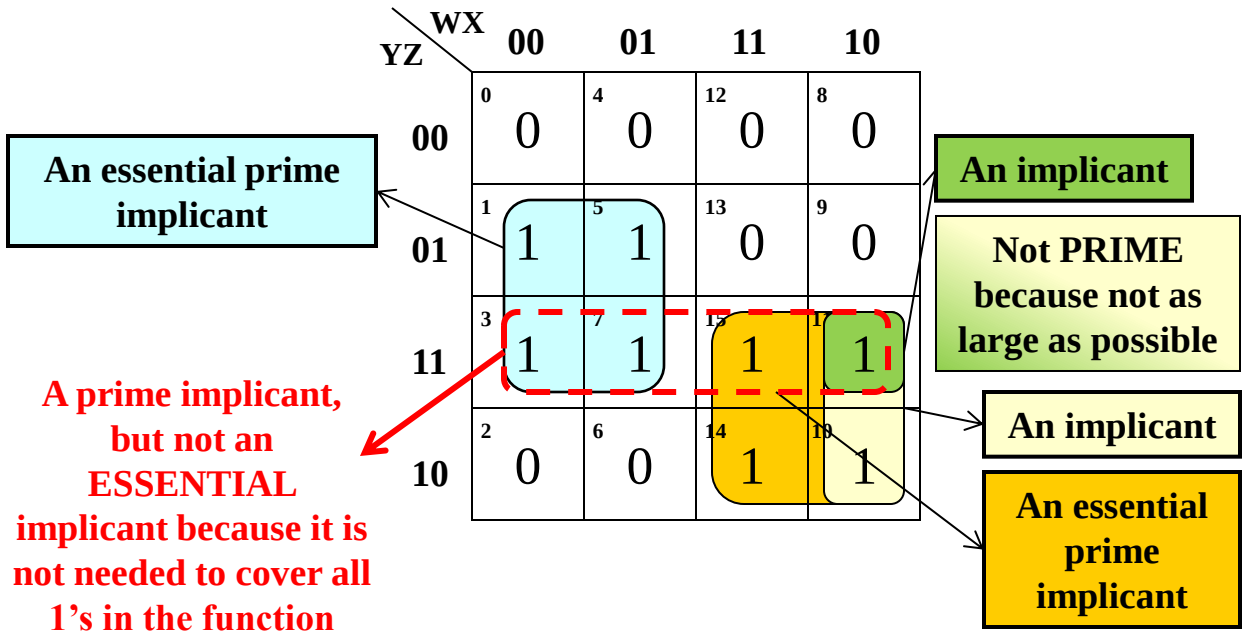
| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **0** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |




**An essential prime implicant (largest grouping possible, that must be included to cover all 1's)**

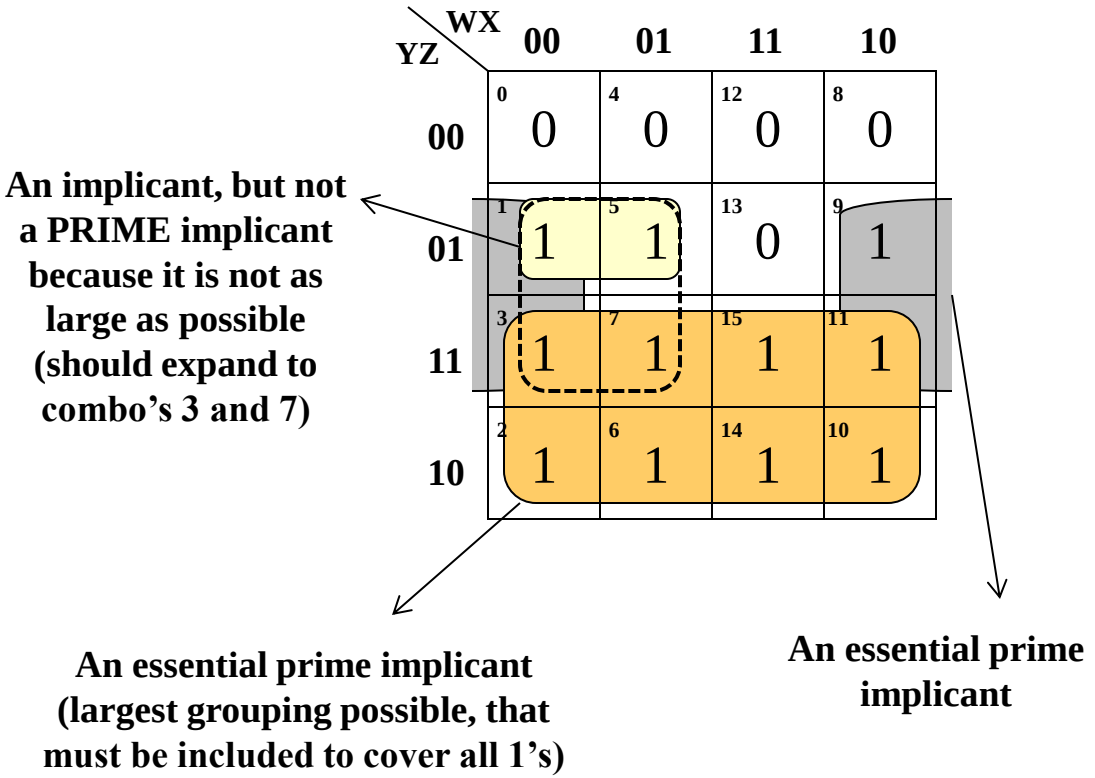
# Implicant Examples

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **0** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **0** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |



# Implicant Examples

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | **0** |
| 0 | 0 | 0 | 1 | **1** |
| 0 | 0 | 1 | 0 | **1** |
| 0 | 0 | 1 | 1 | **1** |
| 0 | 1 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 0 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **0** |
| 1 | 1 | 0 | 1 | **0** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** |



# K-Map Grouping Rules

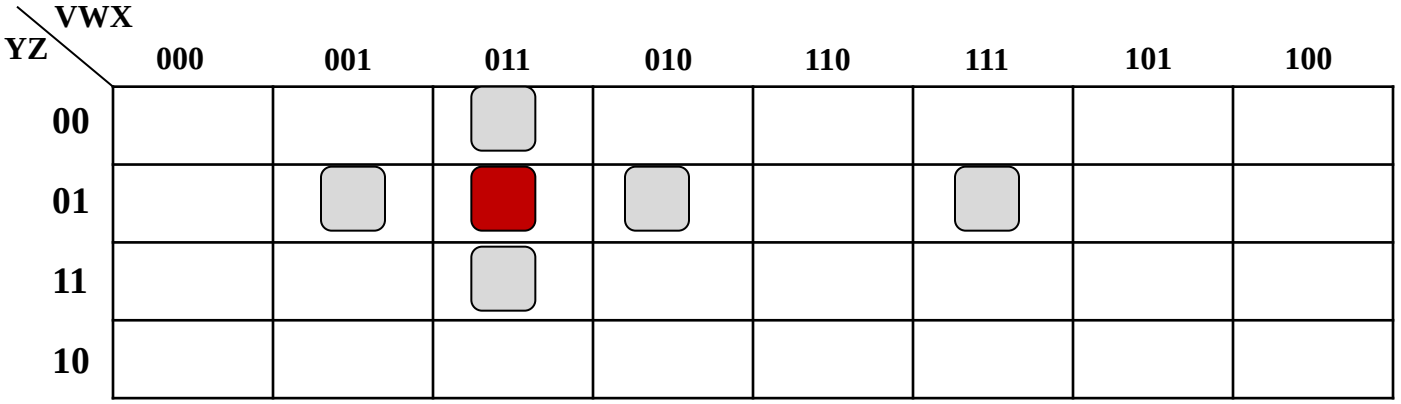
- Make groups (implicants) of 1, 2, 4, 8, … and they must be rectangular or square in shape.
- Include the minimum number of essential prime implicants
  - Use only *essential* prime implicants (i.e. as few groups as possible to cover all 1's)
  - Ensure that you are using ***prime*** implicants (i.e. Always make groups as large as possible reusing squares if necessary)

Informational: You won't be asked to perform 5- or 6-variable K-Maps

# 5- & 6-VARIABLE KMAPS

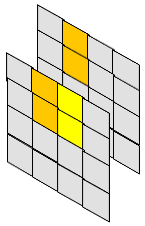
# 5-Variable K-Map

- If we have a 5-variable function we need a 32-square KMap.
- Will an 8x4 matrix work?
  - Recall K-maps work because adjacent squares differ by 1-bit
- How many adjacencies should we have for a given square?
- 5!! But drawn in 2 dimensions we can't have 5 adjacencies.



# 5-Variable Karnaugh Maps

- To represent the 5 adjacencies of a 5-variable function [e.g. f(v,w,x,y,z)], imagine two 4x4 K-Maps stacked on top of each other
  - Adjacency across the two maps



| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 | 0 |
| 01 | 0 | 1 | 1 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |

V=0

These are adjacent

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |

V=1

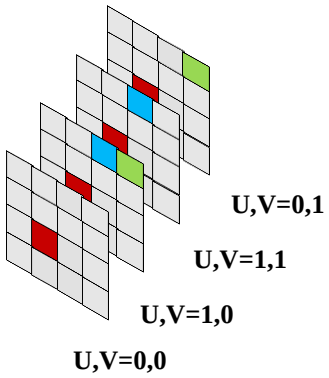
**Traditional adjacencies still apply**
**(Note: v is constant for that group and should be included)**
**=> v'xy'**

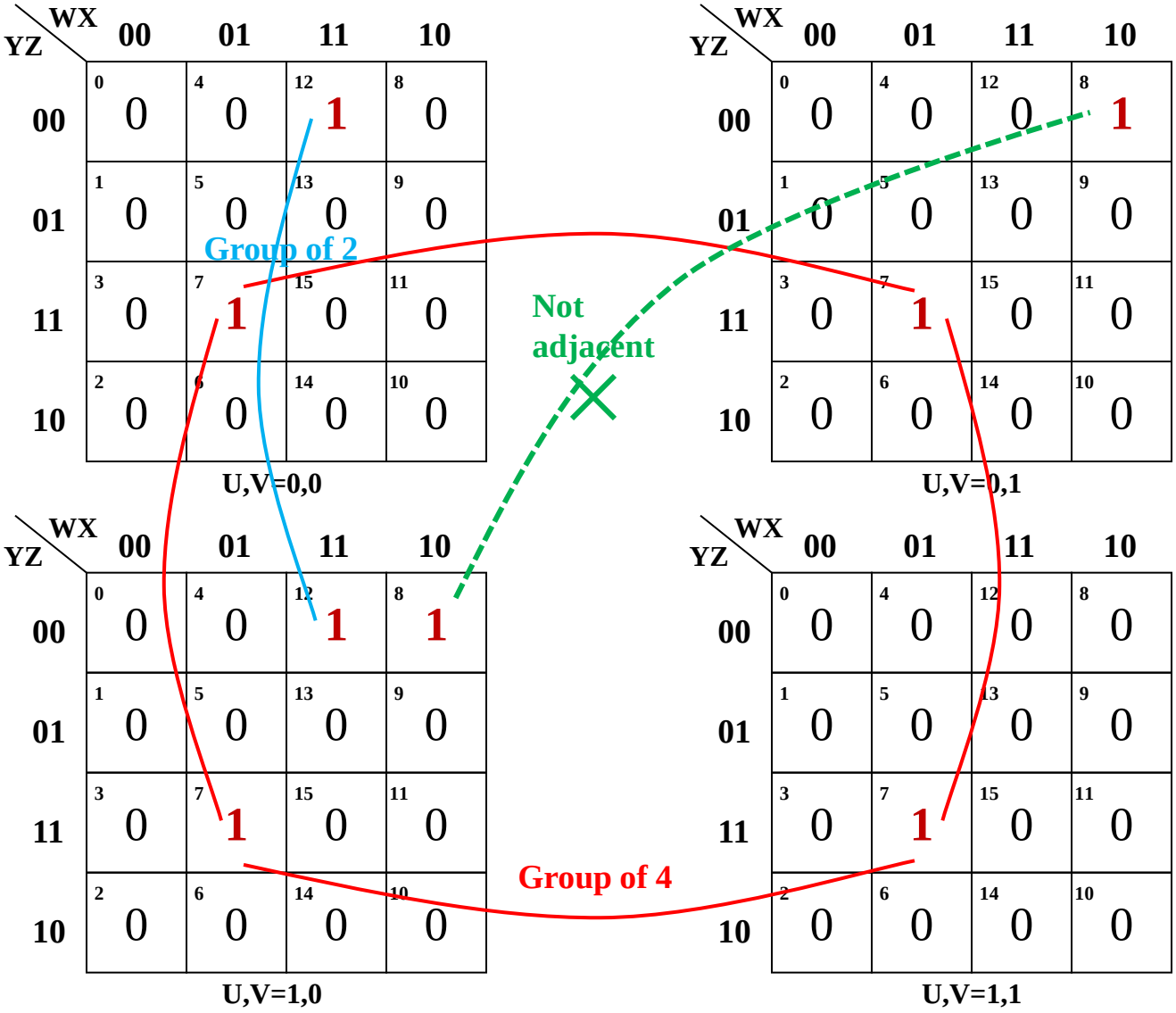
**Adjacencies across the two maps apply**
**(Now v is not constant)**
**=> w'xy'**

$$F = v'xy' + w'xy'$$

# 6-Variable Karnaugh Maps

- 6 adjacencies for 6-variables (Stack of four 4x4 maps)



# 7-Variable K-maps and Other Techniques

- Can we have 7-variable K-Maps?
- No! We would need to see 7 adjacencies per square and we humans cannot visualize 4 dimensions
- Other computer-friendly minimization algorithms
  - Quine-McCluskey
    - Still exponential runtime
    - Minimization is NP-hard problem
  - Espresso-heuristic Minimizer
    - Achieves "good" minimization in far less time (may not be absolute minimal)