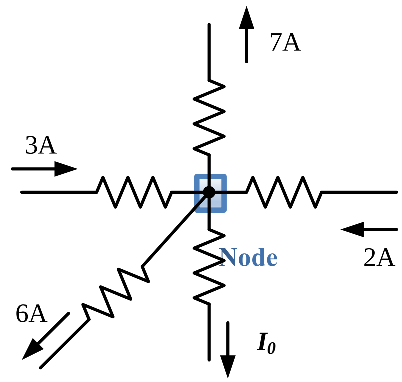
EE 109 Homework 1

Name: ____
Due:

Score: ________

Neatly show your work to get full credit and clearly show your final answer.

1.) **[5 pts.]** Use KCL to solve for I_0.



$$\sum_{i=1}^{n} I_i(node) = 0 \rightarrow \; -3-2+7+6+I_0 = 0$$

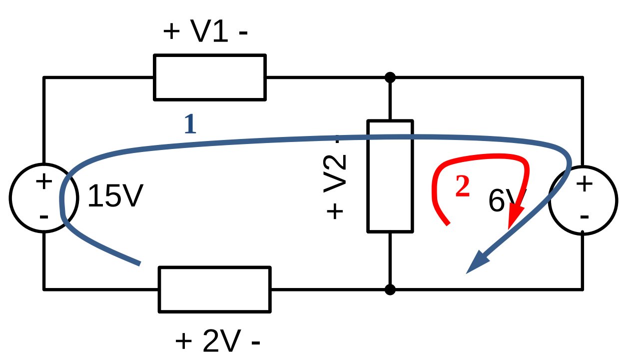
$$\rightarrow I_0 = -8^A$$

Another approach:

$$\sum I_{in}(node) = \sum I_{out}(node)$$

$$\rightarrow \; 3+2 = 7+6+I_0 \rightarrow I_0 = -8^A$$

2.) **[8 pts.]** Use KVL to solve for V1 and V2.

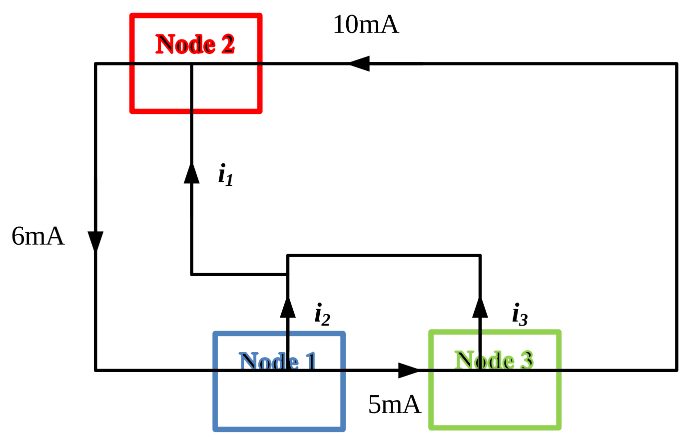


$$\sum_{i=1}^{n} v_i(loop) = 0 \rightarrow$$

$1)\;\; -15 + v_1 + 6 - 2 = 0 \;\; \rightarrow v_1 = 11^v$

$2)\;\; +v_2 + 6 = 0 \rightarrow v_2 = -6^v$

3.) **[9 pts.]** Solve for the currents i_1, i_2, i_3.



$$\sum_{i=1}^{n} I_i(@\,node\ 1) = 0 \rightarrow$$

$$-6+5+i_2 = 0 \;\rightarrow i_2 = 1^A$$

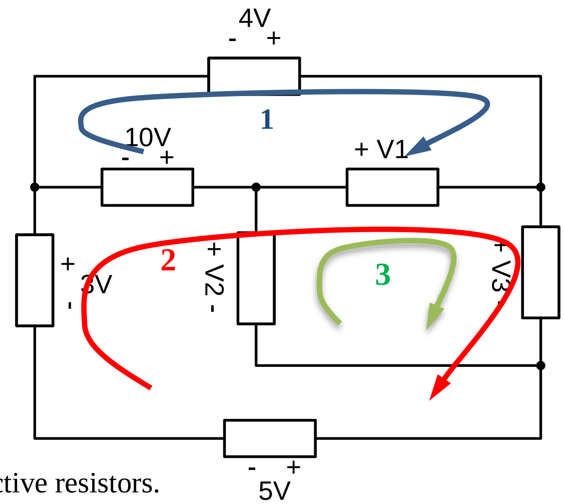
$$\sum_{i=1}^{n} I_i(@\,node\ 2) = 0 \rightarrow$$

$$+6-10-i_1 = 0 \rightarrow i_1 = -4^A$$

$$\sum_{i=1}^{n} I_i(@\,node\ 3) = 0 \rightarrow$$

$$-5+i_3+10 = 0 \;\rightarrow\; i_3 = -5^A$$

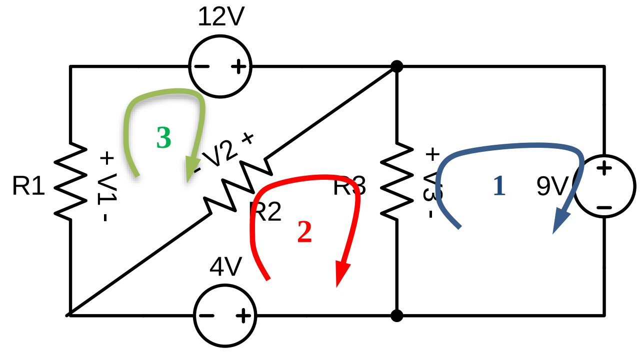
4.) **[9 pts.]** Solve for the voltages V1, V2, V3



$$\sum_{i=1}^{n} v_i(loop) = 0 \rightarrow$$

1) $-10 + v_1 + 4 = 0 \ \ \rightarrow v_1 = 6^v$
2) $-3 - 10 + v_1 + v_3 + 5 = 0 \rightarrow v_3 = 2^v$
3) $-v_2 + v_1 + v_3 = 0 \rightarrow v_2 = 8^v$

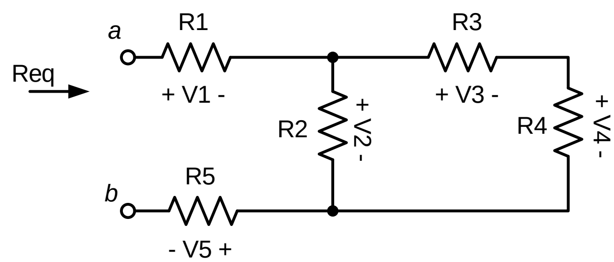
5.) **[9 pts.]** Solve for the voltages V1, V2, V3 across the respective resistors.



$$\sum_{i=1}^{n} v_i(loop) = 0 \rightarrow$$

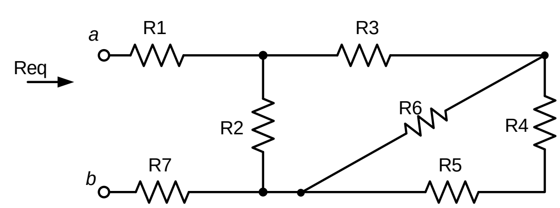
1) $-v_3 + 9 = 0 \ \ \rightarrow v_3 = 9^v$
2) $-v_2 + v_3 + 4 = 0 \rightarrow v_2 = 13^v$
3) $-v_1 - 12 + v_2 = 0 \rightarrow v_1 = 1^v$

6.) **[10 pts.]** Reduce the resistor network shown below to a single equivalent resistance. Leave your answer in terms of R1, R2, R4, R5, and R6.



$$R_{eq} = R1 + [R2 \parallel (R3 + R4)] + R5$$
$$= R1 + \frac{R2.(R3 + R4)}{R2 + R3 + R4} + R5$$
$$= 3 + \frac{4.(4)}{4 + 2 + 2} + 1 = 6\Omega$$

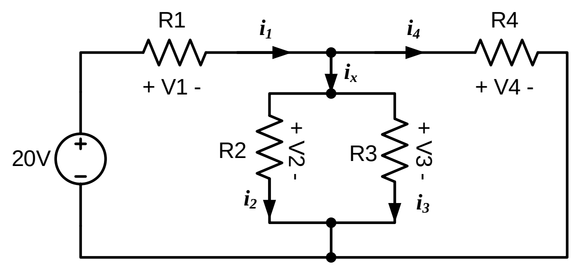
7.) **[10 pts.]** Reduce the resistor network shown below to a single equivalent resistance assuming the following resistor values: R1=5Ω, R2=4Ω, R3=3Ω, R4=1Ω, R5=1Ω, R6=2Ω, R7=7Ω.
Hint: Start by combining R4 and R5 then combine those with R6 and keep going…



$$R_{eq} = R1 + [R2 \parallel (R3 + (R6 \ \parallel (R4 + R5)] + R7$$
$$R_{eq} = R1 + [R2 \parallel (R3 + (2 \parallel (2)] + R7$$
$$R_{eq} = R1 + [R2 \parallel (3 + (1)] + R7$$
$$R_{eq} = R1 + [4 \parallel (4] + R7$$
$$R_{eq} = R1 + [2] + R7 = 5 + 2 + 7 = \mathbf{14\Omega}$$

8.) **[8 pts.]** Find an expression for the current ***i1*** if R1=4Ω, R2=3Ω, R3=6Ω, R4=2Ω.
Hint: Combine R2, R3, R4 into an equivalent resistance which will be in series with R1. From here you can use a KVL loop or Ohm's law to solve for i1.

$$\sum_{i=1}^{n} v_i(loop) = 0 \rightarrow$$
$$-20 + R1.i_1 + (R2 \parallel R3 \parallel R4).i_1 = 0$$



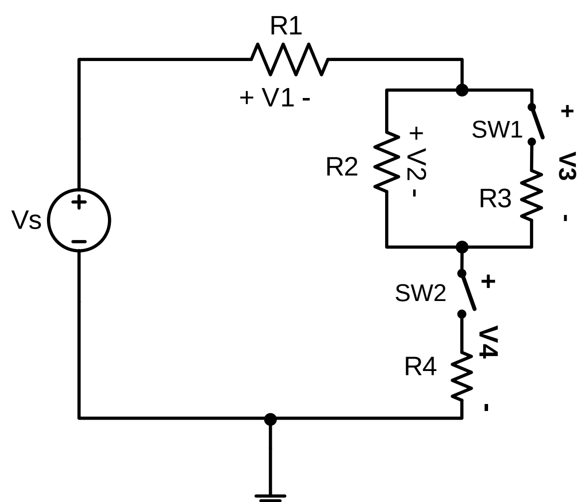
When 3 or more resistors are in parallel you can work with 2 at a time.

$$i_1 = \frac{20}{R1 + \left((R2 \parallel R3) \parallel R4\right)}$$

$$i_1 = \frac{20}{4 + \left((3 \parallel 6) \parallel 2\right)} = \frac{20}{4 + \left(\left(\frac{18}{9}\right) \parallel 2\right)} = \frac{20}{4 + \left(\frac{2 * 2}{2 + 2}\right)} = \frac{20}{5} = 4A$$

9.) **[16 pts.]** Use the generalized concept of a voltage divider (review your notes/lecture slides) to find expressions for the voltage V1 and also V4 in the circuit below. Your expression should be in terms of Vs and R1-R4.

$$i_{R1} = i_{R4} = \frac{v(s)}{R1 + \ R2 \parallel R3 + R4}$$
$$= \frac{v(s)}{R1 + \ R2 \parallel R3 + R4}$$
$$= \frac{v(s)}{R1 + \frac{R2.R3}{R2 + R3} + R4}$$



$$\boldsymbol{V1 = R1 \cdot i_{R1} = R1 \cdot \frac{v(s)}{R1 + \frac{R2.R3}{R2 + R3} + R4}}$$

$$\boldsymbol{V4 = R4 \cdot i_{R4} = R4 \cdot \frac{v(s)}{R1 + \frac{R2.R3}{R2 + R3} + R4}}$$

10.) **[6 pts.]** Look at the circuit from problem 9. If SW2 is open and SW1 closed, what would V4 be (approximately)?

Since SW2 is in series with R4, their resistance can be summed. When a switch is open (not-connected) they can be modeled as an INFINITE resistance. Thus infinity + R4 = infinity.

$$\lim_{R4\to\infty} V4 = \lim_{R4\to\infty} R4.\frac{v(s)}{R1 + \frac{R2.R3}{R2+R3} + R4} = \infty.\frac{v(s)}{\left(R1 + \frac{R2.R3}{R2+R3} + \infty\right)}$$

$$= \infty.\frac{v(s)}{\left(R1 + \frac{R2.R3}{R2+R3} + \infty\right)} = \frac{\infty}{\infty}.v(s) = 1 * v(s) = v(s)$$

11.) **[5 pts.]** Look at the circuit from problem 9. If SW2 is open and SW1 closed, what would V4 be (approximately)?

Since SW1 is in series with R3, their resistance can be summed. When a switch is open (not-connected) they can be modeled as an INFINITE resistance. Thus infinity + R3 = infinity

$$\lim_{R3\to\infty} V4 = \lim_{R3\to\infty} R4.\frac{v(s)}{R1 + \frac{R2.R3}{R2+R3} + R4} = R4.\frac{v(s)}{R1 + \frac{R2.\infty}{R2+\infty} + R4} = R4.\frac{v(s)}{R1 + \frac{R2.\infty}{\infty} + R4}$$

$$= R4.\frac{v(s)}{R1 + R2 + R4}$$

Note: This is the appropriate voltage divider equation had R3 been removed altogether. Thus as a resistance becomes large and is in parallel with others, it's as if it's not even there.

12.) **[5 pts.]** Look at the circuit from problem 9. With SW1 and SW2 closed, if R3 is effectively 0Ω (i.e. replaced by a wire), solve (approximately) for the voltage V4.

$$V4 = R4.\frac{v(s)}{R1 + \frac{R2.R3}{R2+R3} + R4} \to V4(R3 = 0) = \boldsymbol{R4.\frac{v(s)}{R1 + R4}}$$

Note: This is the appropriate voltage divider equation had R2 and R3 been removed completely. Thus as a resistor in parallel becomes small it's as if neither resistor is present.