

CSCI 104L Lecture 10: Introduction to Counting

Question 1. A company has three new employees: Aaron, Gozde, and Hanyuan. There are 12 offices. How many different ways are there to assign offices to these employees?

The Product Rule: If a procedure can be broken down into a sequence of two tasks, and there are n_1 ways to do the first task and n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.

Question 2. How many ways are there to arrange 10 people in a line?

Question 3. How many bit strings of length 8 either start with a 1 or end with two 0's?

The Subtraction Rule: If a task can be done in either n_1 ways or n_2 ways, and there is an overlap between these two methods of n_3 common ways, then the number of ways to do the task is $n_1 + n_2 - n_3$.

Question 4. The company Grinding Gear Games has 350 job applicants. 220 of these applicants are computer science majors, 147 are business majors, and 51 are double majors in computer science and business. How many applicants majored in neither computer science nor business?

Question 5. How many different ways are there to seat 4 people around a circular table, where two seatings are considered identical if each person has the same left neighbor in both seatings, and the same right neighbor in both seatings?

The Division Rule: There are $\frac{n}{d}$ ways to do a task using a procedure which can be done in n different ways, and for any specific way w , it is identical to d of the n total ways.

Question 6. How many ways are there to arrange 10 people in a line, where two lineups are considered identical if they are mirror images of each other?

Question 7. In a version of BASIC, variables can be one or two alphanumeric characters (lower case or capitalized letters are not distinguished). The first character must be a letter. There are 5 two character strings which are reserved and cannot be used. How many different variable names are there?

Question 8. A computer system requires a password between 6 and 8 alphanumeric characters (lower case or capitalization is not distinguished). At least one character must be a digit. How many different passwords are there?

Question 9. How many bit strings of length 4 do not have two consecutive 1's?

Question 10. In how many ways can we select 3 students from a group of 5 students to stand in line for a picture?

A **permutation** of a set of distinct objects is an ordered arrangement of these objects.

An **r-permutation** is an ordered arrangement of r elements of a set. If the set contains n elements, this is denoted as $P(n, r)$.

If n and r are integers with $0 \leq r \leq n$, then $P(n, r) = \frac{n!}{(n-r)!}$

Question 11. How many permutations of the letters DIJKSTRA contain the substring IJK?

An **r-combination** is an unordered arrangement of r elements of a set. If the set contains n elements, this is denoted as $C(n, r)$. This can also be denoted by $\binom{n}{r}$, and called “n choose r”.

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

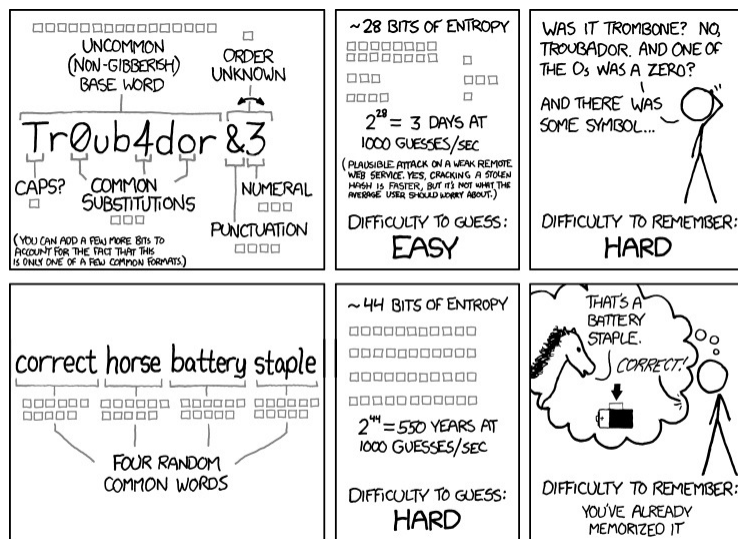
Question 12. There are 7 contestants, and you will select 3 winners (no distinction amongst them). How many different ways are there to do this?

A specific unordered arrangement of r elements can be ordered in $P(r, r) = r!$ ways. Thus $C(n, r) = \frac{P(n, r)}{P(r, r)} = \frac{n!}{r!(n-r)!}$

Question 13. How many different 5-card poker hands can be dealt from a standard deck of 52 cards?

Question 14. How many different 47-card hands can be dealt from a standard deck of 52 cards?

Question 15. How many bit strings of length n contain exactly r 1's?



THROUGH 20 YEARS OF EFFORT, WE'VE SUCCESSFULLY TRAINED EVERYONE TO USE PASSWORDS THAT ARE HARD FOR HUMANS TO REMEMBER, BUT EASY FOR COMPUTERS TO GUESS.

Figure 1: XKCD #936: Password Strength. To anyone who understands information theory and security and is in an infuriating argument with someone who does not (possibly involving mixed case), I sincerely apologize.