Extra Counting Problems

1. How many different binary max-heaps are there that contain exactly the 7 nodes \{0, 1, 2, 3, 4, 5, 6\}?  

2. How many integer solutions are there for the equation: \( x + y + z = 12 \), where \( x, y, z \geq 2 \)?  

3. How many binary search trees have the pre-order traversal \([3, 1, 2, 4, 5, 7]\)? (Hint: this is a trick question)  

4. How many 8 character passwords could you make from the characters \{a-z\}, \{A-Z\}, \{0-9\} (i.e. all the lowercase letters, uppercase letters, and digits) under each of those restrictions:  
   a. There is at least one lowercase letter, one uppercase letter, and one digit.  
   b. There is no restriction at all.  

   Which of the cases, in theory, is more favorable to a hacker?  

5. How many 6 bit strings contain the following pattern: 1010? (examples: 101010, 010100; non-examples: 110110, 001011)