

EE 109 Homework 1

Name: _____

Due: _____

Score: _____

Neatly show your work to get full credit and clearly show your final answer.

1.) [5 pts.] Use KCL to solve for I_0 .

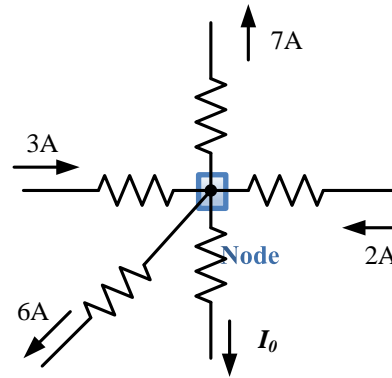
$$\sum_{i=1}^n I_i(\text{node}) = 0 \rightarrow -3 - 2 + 7 + 6 + I_0 = 0$$

$$\rightarrow I_0 = -8^A$$

Another approach:

$$\sum I_{in}(\text{node}) = \sum I_{out}(\text{node})$$

$$\rightarrow 3 + 2 = 7 + 6 + I_0 \rightarrow I_0 = -8^A$$

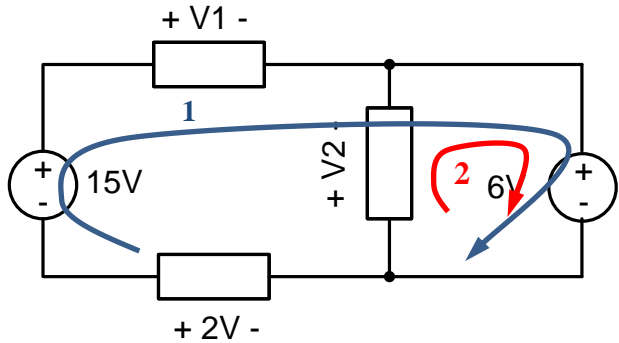


2.) [8 pts.] Use KVL to solve for V_1 and V_2 .

$$\sum_{i=1}^n v_i(\text{loop}) = 0 \rightarrow$$

$$1) -15 + v_1 + 6 - 2 = 0 \rightarrow v_1 = 11^v$$

$$2) +v_2 + 6 = 0 \rightarrow v_2 = -6^v$$



3.) [9 pts.] Solve for the currents i_1 , i_2 , i_3 .

$$\sum_{i=1}^n I_i(@ \text{node } 1) = 0 \rightarrow$$

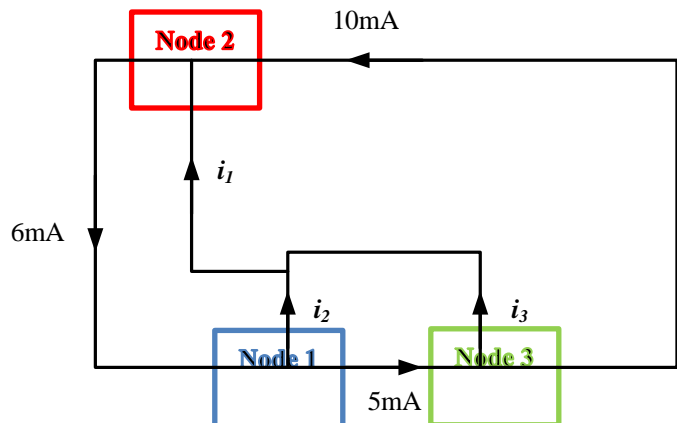
$$-6 + 5 + i_2 = 0 \rightarrow i_2 = 1^A$$

$$\sum_{i=1}^n I_i(@ \text{node } 2) = 0 \rightarrow$$

$$+6 - 10 - i_1 = 0 \rightarrow i_1 = -4^A$$

$$\sum_{i=1}^n I_i(@ \text{node } 3) = 0 \rightarrow$$

$$-5 + i_3 + 10 = 0 \rightarrow i_3 = -5^A$$



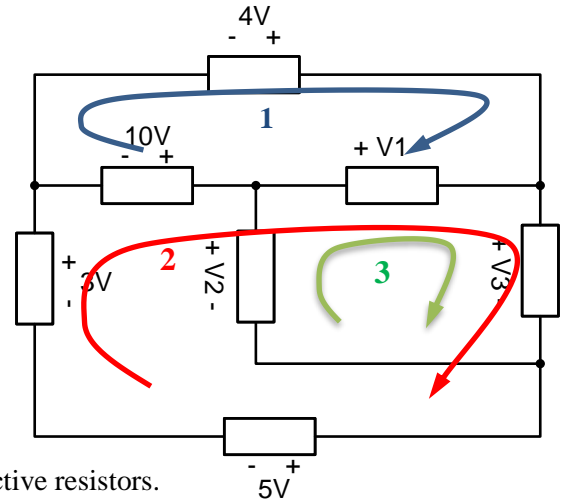
4.) [9 pts.] Solve for the voltages V1, V2, V3

$$\sum_{i=1}^n v_i(\text{loop}) = 0 \rightarrow$$

$$1) -10 + v_1 + 4 = 0 \rightarrow v_1 = 6^v$$

$$2) -3 - 10 + v_1 + v_3 + 5 = 0 \rightarrow v_3 = 2^v$$

$$3) -v_2 + v_1 + v_3 = 0 \rightarrow v_2 = 8^v$$



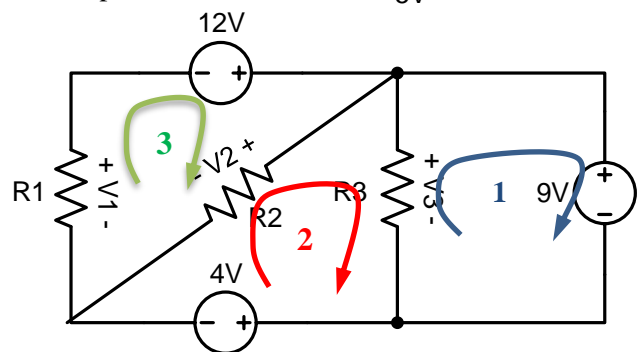
5.) [9 pts.] Solve for the voltages V1, V2, V3 across the respective resistors.

$$\sum_{i=1}^n v_i(\text{loop}) = 0 \rightarrow$$

$$1) -v_3 + 9 = 0 \rightarrow v_3 = 9^v$$

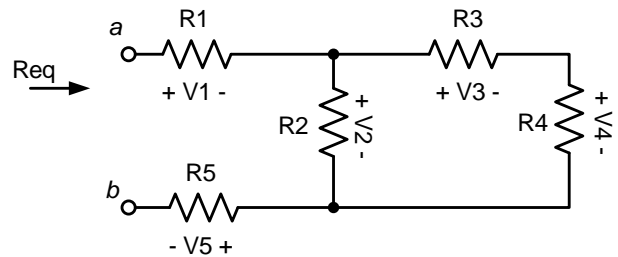
$$2) -v_2 + v_3 + 4 = 0 \rightarrow v_2 = 13^v$$

$$3) -v_1 - 12 + v_2 = 0 \rightarrow v_1 = 1^v$$



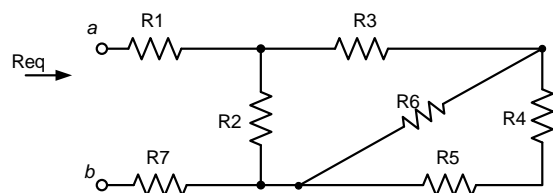
6.) [10 pts.] Reduce the resistor network shown below to a single equivalent resistance. Leave your answer in terms of R1, R2, R4, R5, and R6.

$$\begin{aligned} R_{eq} &= R1 + [R2 \parallel (R3 + R4)] + R5 \\ &= R1 + \frac{R2 \cdot (R3 + R4)}{R2 + R3 + R4} + R5 \\ &= 3 + \frac{4 \cdot (4)}{4 + 2 + 2} + 1 = 6\Omega \end{aligned}$$



7.) [10 pts.] Reduce the resistor network shown below to a single equivalent resistance assuming the following resistor values: R1=5Ω, R2=4Ω, R3=3Ω, R4=1Ω, R5=1Ω, R6=2Ω, R7=7Ω. Hint: Start by combining R4 and R5 then combine those with R6 and keep going...

$$\begin{aligned} R_{eq} &= R1 + [R2 \parallel (R3 + (R6 \parallel (R4 + R5)))] + R7 \\ R_{eq} &= R1 + [R2 \parallel (R3 + (2 \parallel (2)))] + R7 \\ R_{eq} &= R1 + [R2 \parallel (3 + (1))] + R7 \\ R_{eq} &= R1 + [4 \parallel (4)] + R7 \\ R_{eq} &= R1 + [2] + R7 = 5 + 2 + 7 = 14\Omega \end{aligned}$$



- 8.) **[8 pts.]** Find an expression for the current i_1 if $R_1=4\Omega$, $R_2=3\Omega$, $R_3=6\Omega$, $R_4=2\Omega$.
Hint: Combine R_2 , R_3 , R_4 into an equivalent resistance which will be in series with R_1 . From here you can use a KVL loop or Ohm's law to solve for i_1 .

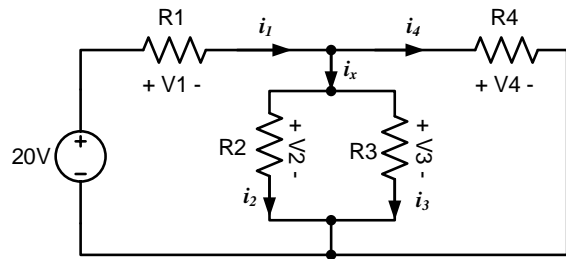
$$\sum_{i=1}^n v_i(\text{loop}) = 0 \rightarrow$$

$$-20 + R_1 \cdot i_1 + (R_2 \parallel R_3 \parallel R_4) \cdot i_1 = 0$$

When 3 or more resistors are in parallel you can work with 2 at a time.

$$i_1 = \frac{20}{R_1 + ((R_2 \parallel R_3) \parallel R_4)}$$

$$i_1 = \frac{20}{4 + ((3 \parallel 6) \parallel 2)} = \frac{20}{4 + \left(\left(\frac{18}{9}\right) \parallel 2\right)} = \frac{20}{4 + \left(\frac{2 * 2}{2 + 2}\right)} = \frac{20}{5} = 4A$$

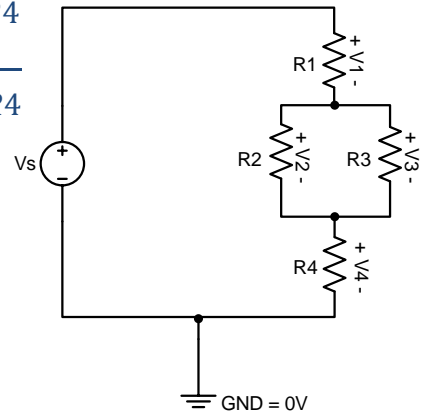


- 9.) **[16 pts.]** Use the generalized concept of a voltage divider (review your notes/lecture slides) to find expressions for the voltage V_1 and also V_4 in the circuit below. Your expression should be in terms of V_s and R_1 - R_4 .

$$i(R_1) = i(R_4) = \frac{v(s)}{R_1 + R_2 \parallel R_3 + R_4} = \frac{v(s)}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} + R_4}$$

$$V_1 = R_1 \cdot i(R_1) = R_1 \cdot \frac{v(s)}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} + R_4}$$

$$V_4 = R_4 \cdot i(R_4) = R_4 \cdot \frac{v(s)}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} + R_4}$$



- 10.) **[6 pts.]** Look at the circuit from problem 9. If R_4 is very large (approaches infinity) what would V_4 be (approximately).

$$\begin{aligned} \lim_{R4 \rightarrow \infty} V4 &= \lim_{R4 \rightarrow \infty} R4 \cdot \frac{v(s)}{R1 + \frac{R2 \cdot R3}{R2 + R3} + R4} = \infty \cdot \frac{v(s)}{\left(R1 + \frac{R2 \cdot R3}{R2 + R3} + \infty\right)} \\ &= \infty \cdot \frac{v(s)}{\left(R1 + \frac{R2 \cdot R3}{R2 + R3} + \infty\right)} = \frac{\infty}{\infty} \cdot v(s) = 1 * v(s) = v(s) \end{aligned}$$

11.) [5 pts.] Look at the circuit from problem 9. If R3 is very large (approaches infinity) again solve (approximately) for the voltage V4.

$$\begin{aligned} \lim_{R3 \rightarrow \infty} V4 &= \lim_{R3 \rightarrow \infty} R4 \cdot \frac{v(s)}{R1 + \frac{R2 \cdot R3}{R2 + R3} + R4} = R4 \cdot \frac{v(s)}{R1 + \frac{R2 \cdot \infty}{R2 + \infty} + R4} = R4 \cdot \frac{v(s)}{R1 + \frac{R2 \cdot \infty}{\infty} + R4} \\ &= R4 \cdot \frac{v(s)}{R1 + R2 + R4} \end{aligned}$$

Note: This is the appropriate voltage divider equation had R3 been removed altogether. Thus as a resistor in parallel get's large, it's as if it's not even there.

12.) [5 pts.] Look at the circuit from problem 9. If R3 is effectively 0Ω (i.e. replaced by a wire), solve (approximately) for the voltage V4.

$$V4 = R4 \cdot \frac{v(s)}{R1 + \frac{R2 \cdot R3}{R2 + R3} + R4} \rightarrow V4(R3 = 0) = R4 \cdot \frac{v(s)}{R1 + R4}$$

Note: This is the appropriate voltage divider equation had R2 and R3 been removed completely. Thus as a resistor in parallel get's small it's as if neither resistor is present.