Final Review Lab!

Congrats!!! You are almost done with 104!

***disclaimer, these slides only cover content not on past review labs*

TIPS

EXAM TIME: **WEDNESDAY, MAY 1ST, 8AM**

- **Start your cheat sheet early! Potential things to include:**
  - Counting/Probability Formulas
  - ADTs and their runtimes based on implementation
  - Recursion/backtracking steps (ex. Base case, recursive step, “undoing” step)
  - Anything you have been struggling with!
- **Go through notes, labs, programming assignments, midterms, and practice exams**
- **Try to come up with your own practice problems**
- **Get enough sleep!**

Counting

Important Rules:

- Product rule:
  - If procedure can be broken up into sequence of k tasks
  - n1 ways to do first task, n2 ways to do second task, nk ways to do kth task
  - **n_1 * n_2 * … * n_k ways to do the procedure**

- Sum rule:
  - If procedure can be done in n_1 ways OR n_2 ways
  - n_1 and n_2 have zero overlap
  - **n_1 + n_2 ways to do the task**

Counting

Important Rules (cont.):

- Subtraction rule:
  - If procedure can be done in n_1 ways OR n_2 ways
  - n_1 and n_2 have overlap n_3
  - **$n_1 + n_2 - n_3$ ways to do the task**
- Division rule:
  - If procedure can be done in n ways
  - For each way, it is identical to d-1 other ways
  - **n/d ways to do a task**

Counting

Important Rules (cont.):

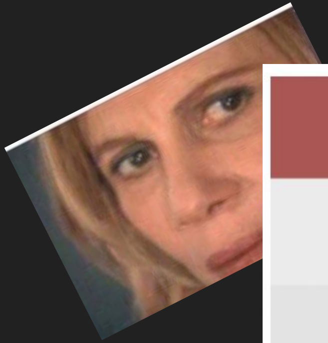
- Permutation: **ordered** arrangement of r elements from a set of n

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

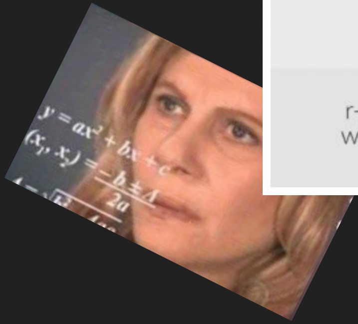
- Combination: **unordered** arrangement of r elements from a set of n (n choose r)

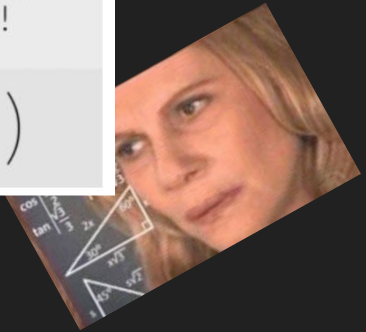
$$ {}_nC_r = \binom{n}{r} = \frac{{}_nP_r}{{}_rP_r} = \frac{n!}{r!\,(n-r)!} $$

Counting



	Does order matter?	Is repetition allowed?	Formula
r-permutation without repetition	Yes	No	$\frac{n!}{(n-r)!}$
r-permutation with repetition	Yes	Yes	n^r
r-combination without repetition	No	No	$\frac{n!}{r!\,(n-r)!}$
r-combination with repetition	No	Yes	$\binom{n-1+r}{r}$





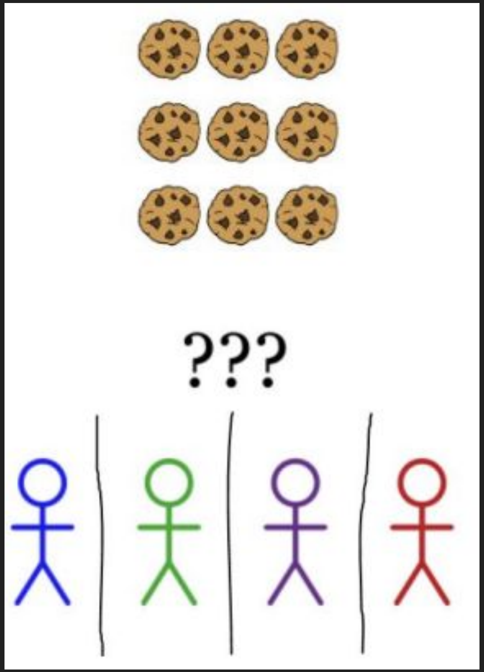
Counting

How many different ways are there to distribute 9 cookies to 4 children so that each child gets at least one cookie?

Counting

- The cookies are indistinguishable while the children are distinguishable, so Stars and Bars is a good option to use.
    - 9 stars = cookies
    - 3 bars to separate children

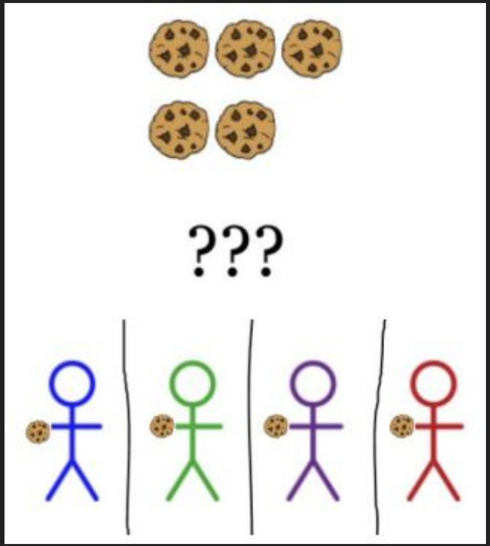
- Problem: Using the Stars and Bars equation will give sequences with children getting no cookies.
    - How do we make sure each child gets at least one cookie with this method?



Counting

- We can give each child a cookie, leaving us with 5 cookies (stars) remaining to distribute.

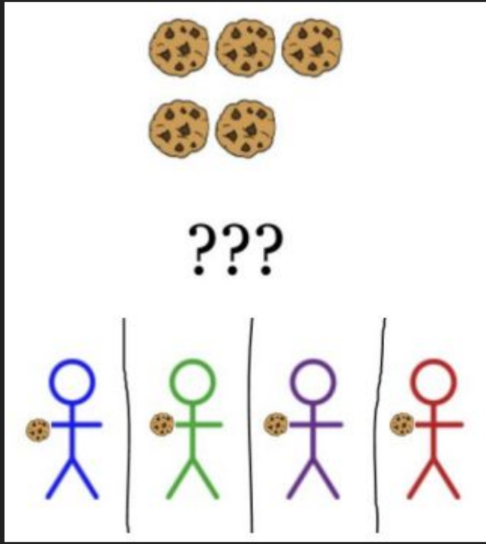
- More importantly, we now have no restrictions on how to distribute those 5 cookies since the given condition will always be fulfilled.



Counting

- We now have no restrictions on how to distribute those 5 cookies since the given condition will always be fulfilled.

- Now just need to use with the remaining 5 cookies and 3 bars, giving us:
[(5 + 3) CHOOSE 3] = **56 ways**



Counting

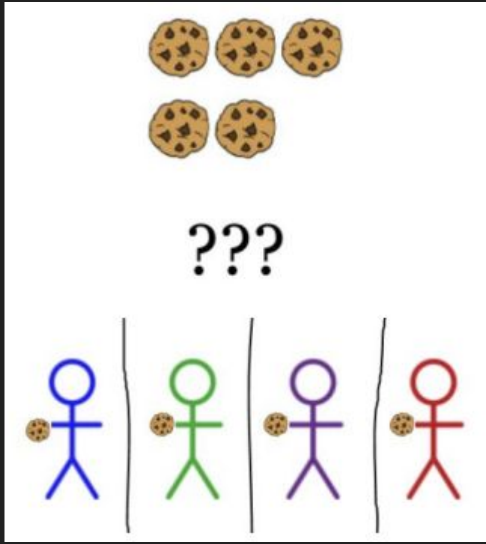
Now let's say the children are indistinguishable. How many different ways are there to distribute 9 cookies to 4 children so that each child gets at least one cookie?

Counting

- We will still give each student one cookie, so now we need to distribute 5 indistinguishable cookies over the 4 indistinguishable students.

- There is no formula for this situation.
  - (5, 0, 0, 0)
  - (4, 1, 0, 0), (3, 2, 0, 0)
  - (3, 1, 1, 0), (2, 2, 1, 0)
  - (1, 1, 1, 2)

- Total: **6 ways**



Probability

Important Rules:

- Probability of event E:
  - S = sample space of equally likely outcomes
  - **P(E) = |E| / |S|**

- Complement: probability that event does not occur
  - **$P(\bar{E}) = 1 - P(E)$**

Probability

Important Rules:

- Conditional Probability: probability of B given A

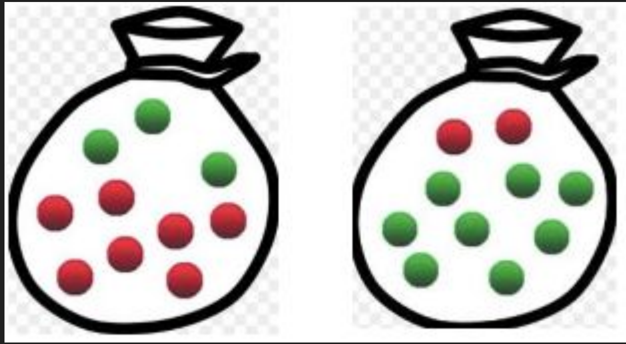
$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

  - If likelihood of B occurring does not depend on A, then B is independent of A:

$$P(B \mid A) = P(B).$$

Probability

- Suppose there are two bags in a box, which contain the following marbles:
  - Bag 1: 7 red marbles and 3 green marbles.
  - Bag 2: 2 red marbles and 8 green marbles.



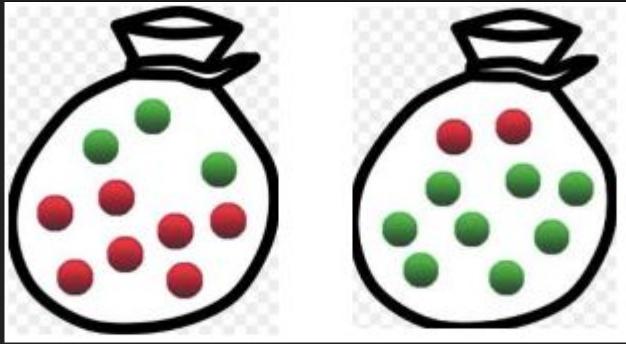
- If we randomly select one of the bags and then randomly select one marble from that chosen bag, what is the probability that it's a green marble?

Probability

- Green marble could come from Bag 1 or Bag 2, which will affect the chances of drawing a green marble

- We need to use the Law of Total Probability:
    - For any partition of the sample space into disjoint events $F_1, \ldots, F_k$:

      $$\mathbf{p(E) = p(E|F_1) * p(F_1) + \ldots + p(E|F_k) * p(F_k)}$$



Probability

Important Rules:

- Probability of event E:
  - S = sample space of equally likely outcomes
  - **P(E) = |E| / |S|**

- Complement: probability that event does not occur
  - **$P(\bar{E}) = 1 - P(E)$**

Probability

Important Rules:

- Conditional Probability: probability of B given A, Bayes Theorem

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

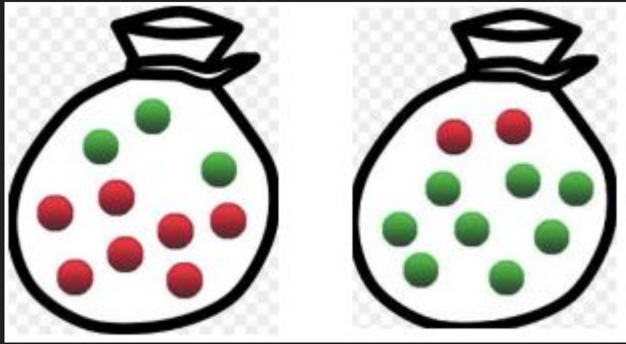
$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

- If likelihood of B occurring does not depend on A, then B is independent of A:

$$P(B \mid A) = P(B).$$

Probability

- Suppose there are two bags in a box, which contain the following marbles:
  - Bag 1: 7 red marbles and 3 green marbles.
  - Bag 2: 2 red marbles and 8 green marbles.



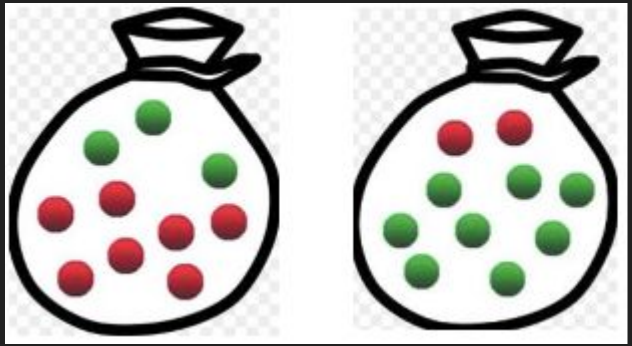
- If we randomly select one of the bags and then randomly select one marble from that chosen bag, what is the probability that it's a green marble?

Probability

- Green marble could come from Bag 1 or Bag 2, which will affect the chances of drawing a green marble

- We need to use the Law of Total Probability:
  - For any partition of the sample space into disjoint events $F_1, \ldots, F_k$:

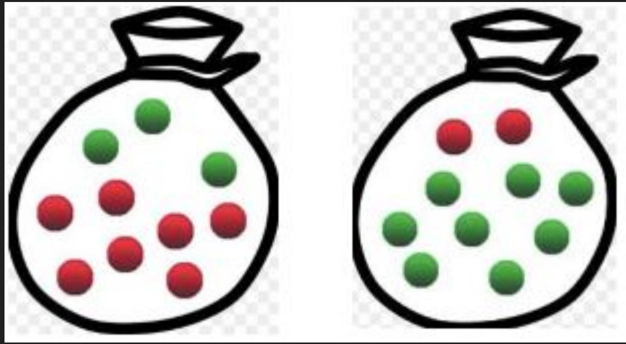
    $$\mathbf{p(E) = p(E|F_1) * p(F_1) + \ldots + p(E|F_k) * p(F_k)}$$



Probability

$p(G) = p(G|B_1) * p(B_1) + p(G|B_2) * p(B_2)$

- $p(B_1) = p(B_2) = 0.5$
- $p(G|B_1) = 3 / (7+3) = 3/10 = 0.3$
- $p(G|B_2) = 8 / (8+2) = 8/10 = 0.8$



$p(G) = (0.3 * 0.5) + (0.8 * 0.5)$

$p(G) = 0.15 + 0.40$

$p(G) = \mathbf{0.55}$

Number Theory

Q: Say gcd(a, b) = 1 and gcd(a, c) = 1. What is gcd(a, b*c)?

Number Theory

Q: Say gcd(a, b) = 1 and gcd(a, c) = 1. What is gcd(a, b*c)?

A: **1**. Think about breaking a, b, and c into their prime factors. Since a is coprime with both b and c, when we multiply them together, we don’t gain any factor in the product that will magically make the gcd greater than 1!

Number Theory

Q: Is 257 prime?

Number Theory

Q: Is 257 prime?

A: sqrt(257) = 16 (roughly)

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 → prime!

Number Theory

Q: What is the ones digit of 7^{100}?

Number Theory

Q: What is the ones digit of 7^100?

A: Let's try to find a pattern…

7^ 0 = 1

7^1 = 7

7^2 = 49 = 9

7^3 = 343 = 3

7^4 = 2401 = 1

7^5 = 16807 = 7

7^6 = 117649 = 9

Number Theory

Q: What is the ones digit of 7^100?

A: Pattern repeats in groups of 4: 1 7 9 3 → 1 7 9 3 → etc.

Which number is 100 in the pattern? (i.e. will it be 7 9 3 or 1?)

N = exponent

If n % 4 == 0, last digit 1

If n % 4 == 1, last digit 7

If n % 4 == 2, last digit 9

If n % 4 == 3, last digit 3

N = 100 → 100 % 4 = 0, **last digit is 1**

Number Theory

Q: Given that $5x \equiv 6 \pmod{8}$, find x.

Number Theory

Q: Given that $5x \equiv 6 \pmod{8}$, find x.

A: This means that 6 % 8 == 5x % 8

5x % 8 = 6. Need to find an x that will satisfy this.

Go through multiples of 5, think if remainder 8 == 6

5, 10, 15, 20, 25, 30

X = 6

Bloom Filters

Q: What are the benefits and drawbacks of using a bloom filter?

Bloom Filters

Q: What are the benefits and drawbacks of using a bloom filter?

A:

Benefit: avoids storage of keys (better space efficiency wise)

Drawback: Can be space inefficient if not implemented correctly, not always right…

Bloom Filters

Q: Which is possible, false positives or false negatives?

Bloom Filters

Q: Which is possible, false positives or false negatives?

A: **False positives!** We may accidentally say something exists in the hashtable if all bits are set, but we'll never accidentally say something is not there when it is

Why?

Because we would have set all the bits if we were inserting the item in the first place!

Bloom Filters

Q: Let’s say we have a bloom filter with 19 indices, 3 universal hash functions. 5 of the bits are set. What is the probability of getting a false positive?

Bloom Filters

Q: Let’s say we have a bloom filter with 19 indices, 3 universal hash functions. 5 of the bits are set. What is the probability of getting a false positive?

A: Product rule, since the problem can be broken up into 3 “tasks”

hash function 1, 2, 3: all 5/19 chance of hitting set bit

5/19 * 5/19 * 5/19 = 1.8% chance!